A Joint Response Model for Matched Decision Makers: Exploring Decision Making Mechanism for Mutually-selected Agents

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ABSTRACT

Burgeoning information technology innovations and the wide adoption of GPS devices have greatly changed the transportation system. For travelers, the real-time ridesharing platforms (e.g., Uber) allow drivers and riders to interact, pair up, and jointly decide on departure time and routes. In freight transportation, the prosperity of e-commerce leads to individualized real-time seller-buyer matching and their joint decisions on delivery modes and time windows. Transportation agents mutually select, or get matched with their counter-partners, and jointly make decisions on a set of matters that can be measured as linear, ordinal, or categorical values. Popular and potential methodologies of understanding these emerging collaborative phenomena include agent-based modeling, cooperative game theory, optimization-based approaches, and econometric modeling. Among these methods, econometric modeling does not assume any behavioral rules and allows the collected data to elucidate this matter. However, existing econometric models are not able to behavioral-consistently capture these new phenomena, such as intricate matching network, mutual selection, and intensive joint decision making. Therefore, this dissertation develops an innovative econometric model to fill the void. Specifically, the proposed model consists of two parts: The first part explains the matching process in a many-to-many matching structure; The second part characterizes the joint decision making process of mutually-selected decision makers. The two parts are integrated by recognizing their dependency that is essentially a sample selection process: a joint response is only observed for matched decision makers. The proposed model is estimated using a Bayesian Markov-Chain Monte-Carlo approach with data augmentation. The likelihood functions and posterior distributions are derived for the ordinal and multinomial joint response outcomes respectively. Then, a simulation dataset is generated based on pre-defined parameters, and parameter recovery capability is measured as an indicator of model performance. A series of simulation datasets are further generated with respect to different parameter settings to evaluate the sensitivity of parameter recovery capability. Lastly, two empirical transportation applications are presented to demonstrate applicable values of the proposed model. The first application investigates flight on-time performance considering the mutual selection and joint responses of airlines and airports. The second application analyzes

freight carriers' responses to hypothetical toll increases with the consideration of their interactions with freight customers.

Key words: matching problem; joint decision making; Bayesian inference; airline-airport vertical relationship; freight transportation

1. Introduction

In the modern human history, collaboration has greatly contributed to the development of transportation systems. The concept of collaboration is broad: any activity that is conducted jointly by multiple people can be collaboration. This dissertation focuses on a narrower definition of collaboration, which only conceptualizes activities of mutual selection and joint decision making for decision makers in a two-side market. In transportation, this narrow concept of collaboration can be observed between regional public agencies on infrastructure construction projects, drivers and passengers on public transportation, as well as suppliers and customers in freight deliveries. In recent years, burgeoning information technology innovations and the wide adoption of global positioning system (GPS) devices have brought new characteristics to this narrower definition of collaboration, which feature intricate matching networks, mutual selection, and intensive joint decision making processes. These new characteristics reshape transportation patterns in terms of each individual's trip distance, frequency, mode/route/time choices and consequently, network traffic, emission, and sustainability. Insufficient analysis of these new characteristics would prevent the research community and practitioners from fully understanding the emerging transportation phenomena. This chapter will first introduce the behavioral background of collaboration, followed by a discussion of motivation and statement of problems. Finally, objectives of this dissertation will be presented.

1.1 Background

Collaboration becomes increasingly crucial for the transportation system with the development of travel modes and the expansion of road networks. The very first means of transportation did not require collaboration. People could walk by foot, use animal-pulled vehicles, and take advantage of simple tools without the help of other people. After the 17th century, collaboration started to be observed widely in moving people and goods between locations. For example, mails were delivered by relays of mailmen. Drivers were employed to maneuver trains, automobiles, and aircrafts for passengers to achieve destinations. In recent decades, collaboration could be observed in even more aspects of transportation: transportation departments worked together on infrastructure construction projects. Multimodal coordination was conducted across agencies to smoothly move travelers from one mode to the other.

Classic collaborative activities in transportation share some common characteristics: (1) The collaborators' matching network is simple. For example, a taxi driver may carry only one rider or one group of riders at a time (e.g., a one-to-one matching structure); (2) The collaborative relationship is determined by only one side: Taxi drivers are not able to proactively select riders but accept riders' choices; and (3) Joint decision making is not frequent. The only joint decision made in a taxi trip may be the trip route.

These collaboration patterns are changing along with burgeoning information technology innovations. A typical example of such innovations includes fast-evolving e-commerce businesses, which enabled frequent small-package deliveries between sellers and buyers, such as the merchandise supported by Craigslist. Another example is the mobile app-based taxi service, such as Uber, which provides real-time driver-rider matching. These emerging collaborations present certain characteristics that have not been observed in classic collaborations. This dissertation summarizes the new trend as intricate matching networks, mutual selection, and intensive joint decision making processes. The proposed joint response model will be developed based on the understanding of these emerging characteristics.

1.2 Motivation and Statement of Problems

The emerging characteristics in collaboration cannot be analyzed sufficiently by existing methodologies. Throughout the literature, collaboration is mostly investigated by agent-based modeling and game theory approaches where the main objective is to understand decision outcomes by simulating decision makers' behavior. Dale Mortenson was honored by the Nobel Prize in Economics in 2010 for his work in market search frictions (A summary can be found in Mortensen (2011)), which explicitly focused on how a collaboration relationship is established. Collaboration is also investigated using optimization-based models where the focus is on finding optimal solutions of each decision maker or the entire system. These models have to pre-define certain behavioral rules while econometric modeling does not assume these rules and allows collected data to elucidate the formation and outcome of collaboration. However, the econometric literature of collaboration is far from comprehensive. Although a few papers, which will be reviewed in detail in the Literature Review section, have touched the problem of collaboration, immense room exists in improving existing works both conceptually and methodologically. For example, typical travel demand models assume a travel activity is determined by only one traveler rather than collaborations of multiple decision makers. This assumption excludes the possibility of interaction and joint decision making on travel behavior. However, this assumption fails in many situations, especially freight transportation, where cargos are delivered upon the agreement of multiple decision makers. Travel demand models with respect to multiple decision makers are mainly discussed in a context of intra-household interaction. These studies assume that the grouping of multiple decision makers is pre-determined, which forbids the possibility of mutual selection in the joint decision making process. As for analyzing mutual selection and joint decision making, the most relevant works are the econometric matching model. This immature topic has a limited number of studies that use empirical data to understand agents' joint behavior. Among them, Sorenson (2007) conducted a representative work to investigate firms' initial public offerings using a one-to-many matching network, binary decision outcome, and a restricted variance-covariance matrix in understanding the dependency of mutual selection and joint decision processes. Enlighted by his work, this dissertation develops an econometric matching model to investigate a many-to-many matching network, ordinal/multinomial decision outcomes, and a flexible variance-covariance matrix. Collectively, this dissertation is motivated by the importance and insufficiency of research in understanding agents' joint behavior to develop an innovative econometric model that behavior-consistently models the mutual selection and joint decision making processes.

The next few subsections explain the new characteristics in collaboration, respectively, using an example of supplier-customer interaction, and introduce the merit of the proposed model.

1.2.1 Intricate Matching Network

The matching network of transportation agents can be complicated. The matching network can be observed for a one-side market: a supplier can match with other suppliers. This type of matching may result in competition where agents strive to improve their respective market shares. It can also lead to collaboration where agents share capacity and access to goods and service. Another type of connection occurs in a two-side market, which usually enables collaboration exclusively. In a two-side market, the matching network has a set of matching structures: one-to-one matching, one-to-many matching, and many-to-many matching. This dissertation will focus on the many-to-many matching because this is the most complicated structure for a two-side market. Note that the matching structure can be further extended to involve multiple sides. However, such a complex network likely results in lengthy computation and infeasible solutions given the existing computation machine so that it is left for future works.

In a two-side market, an agent can be connected with all agents on the other side. Taking each state in the U.S. as the agent, trade can be observed between any two states. This all-to-all matching structure is pre-determined and mostly observed in the aggregate analyses, such as migration, trade, and traditional travel demand analysis. In many disaggregate-level analyses, most agents are not able to match with all counterparties so that the connection with a proportion of agents is often observed. The formation of this sort of network can be attributed to a mutual selection process: both agents have to content each other so that a connection is established. This additional reasoning step leads to an important objective of this dissertation, which is to explain the mutual selection process and explain its effect on the joint response.

In a short summary, the investigated matching network of a two-side market comprises multiple agents on both sides with each agent connecting with multiple counterparties.

1.2.2 Mutual Selection

Mutual selection is a process that agents on both sides choose their best counterparties. Using the supplier-customer interaction as an example, a supplier starts with assessing the characteristics of all customers in the market and, based on the assessment, selects the customers that best fulfill the supplier's desire. The characteristics assessed include how much revenue the customer could bring to the supplier, customer's credibility, and customer's business strategies, etc. Sometimes, the supplier's most favorable customers might dislike the supplier and consequently, the supplier has to turn to the next best customer until the desired customer also likes the supplier. As the partner selection process is bidirectional, the same assessment process occurs on the customer's side at the same time. The customer assesses all suppliers in the market in terms of their potential cost, reliability, and speed of delivery. Based on the assessment, the customer proposes to the best supplier. If rejected, the customer proposes to the next best supplier. A supplier and a customer can finally be matched only if both are happy with each other. Similar bidirectional partner selection processes can be widely observed in marriage, college admission, and business coalition problems.

Characteristics assessed by the counterparty include not only attributes of the target agent, but also joint attributes determined by both sides. For example, a supplier assesses not only the targeted customers' industry sectors, but also spatial proximity between the supplier and customer, collaborative history, and debt payable conditions between them. In transportation, the most interesting joint attribute is spatial proximity, which uses Euclidean distance, network distance, travel time, or any other meaningful measurements to characterize the nearness in space. The spatial proximity can be also measured by contiguity, i.e., whether two regions are contiguous with each other. Coalition is another alternative for understanding the nearness of business entities.

The proposed models identify the unknown parameters using observed matching data, which is the observed matching network: an agent on one side has multiple matched and unmatched pairs on the other side. Such matched and unmatched relationships imply a series of inequality conditions of pairwise utility (e.g., the preference of matching). The absolute value of pairwise utility is of no interest but the relative magnitude enables the possibility to identify parameters in the model. The use of relative utility can be perceived as identifying to a special type of limited dependent variable models. The estimation results will reveal the attribution of influential factors on the formation of matching.

1.2.3 Joint Decision Making

A joint decision is reached based on common interests and compromise of conflicting claims between matched decision makers. Without considering partners, each agent could fulfill his own desire. However, such a decision may not be accepted by the counterparty and consequently, efforts are recognized in reaching an agreement by adjustment of different claims. As a result, a popular method of studying joint decision making is to simulate the negotiation process in reaching an agreement, such as agent-based modeling and cooperative game theory. From the perspective of econometric modeling, conflicting claims are captured by taking both sides' attributions into consideration. The joint decision outcome is a result of reconcilement of contributions from each side.

The outcome of a joint decision can be continuous, ordinal, and categorical. In freight transportation, typical continuous outcomes are cargo weight, gas emission, and travel frequency. Travel frequency can be also treated as an ordinal outcome (e.g., infrequent, moderately frequent, and frequent). Categorical outcomes include mode/route/time choices. The difference of outcome values enables different specifications of econometric models: regression models that correspond to the three types of outcomes are the standard regression, the ordinal discrete response model, and the multinomial discrete outcome model. The proposed joint response model will be able to analyze all of the three types of outcomes, but with an emphasis on the discrete outcomes.

The relationship of the joint decision making and the mutual selection process is analogous to a sample selection process: the joint decision outcome is only observed for matched pairs (e.g., a subset of entire sample). If the sample selection process is not taken into account, the joint decision would likely end up with biased estimation. A typical sample selection model usually consists of two equations with one binary outcome equation identifying the subsamples and the other linear model analyzing outcomes in interest. The error terms of the two equations are assumed to be correlated with each other and estimated by empirical data. If the correlation is found insignificant, the sample selection model reduces to two separate equations, which is named the two-part model. The proposed model can be perceived as an extension to the classic sample selection model: the first equation replaces the binary outcome model with the matching equation. The second equation will be extended from a standard linear regression to an ordered probit model and a multinomial probit model to capture discrete outcomes. The error terms will be specified in a similar sample selection manner, but some extensions will be made to incorporate higher-dimension correlation structure in the multinomial case. Therefore, from the perspective of sample selection models, the proposed models contribute to the literature by using a matching equation to analyze the sample selection process and specifying discrete outcome models to investigate discrete outcomes.

1.3 Objectives

The main objective of this dissertation is to develop an innovative joint response model to analyze new trends of collaboration in transportation. The proposed model will be analyzed in terms of its underlying behavioral background, mathematical specification, validation and sensitivity analysis, and empirical applications.

The underlying behavioral background focuses on consistently describing the collaboration data generating process. In specific, the process can be understood based on the queries of (1) which two agents are matched with each other in a two-side market, (2) how matched agents make joint decisions, and (3) how the matching process and decision making process interact with each other.

The proposed model will be specified by understanding the discussed data generating process. Conditions of pairwise utility will be inferred to disentangle the intricate matching network in the matching equation. The joint decision making equation will analyze discrete joint decision outcomes and connect with the matching equation in a sample selection manner. As any one pairwise utility is dependent on the pairwise utility of all other possible pairs, traditional maximum likelihood estimation would encounter the difficulty of maximizing a high-dimensional integral. This dissertation avoids such a computational complex by using a Bayesian Markov-Chain Monte-Carlo (MCMC) simulation with data augmentation approach to estimate the parameters of interest.

The estimation approach will be validated using simulation data. The steps include (1) define the values of parameter; (2) randomly generate simulation data based on the values of pre-defined parameters; (3) use the Bayesian MCMC approach to estimate the parameters; and (4) compare the estimated values and the pre-defined values of parameters. If the estimated values are close to the pre-defined values, the estimation approach is validated. Furthermore, simulation with respect to different parameter settings will be studied to test the sensitivity of the proposed estimation approach.

Lastly, two empirical applications will be presented using the proposed model to demonstrate the applicable values in practice. The first application investigates the airline-airport collaboration in measuring flight on-time performance. The second application analyzes the freight carrier-customer collaboration on their joint responses to hypothetical toll increases.

The major contributions of this dissertation to the existing literature can be summarized as follows. This dissertation

- Extends matching models to analyze a many-to-many matching structure, ordinal/multinomial discrete outcomes, and a flexible covariance setting.
- Extends sample selection models by using the matching result.
- Improves the understanding of joint behavior in a sharing economy.
- Provides important insights into the formation and joint response of airline-airport vertical collaboration.
- Adds important values in understanding freight agents' interactions by econometric modeling.

2. Literature Review

Burgeoning technology innovations and fast popularized information exchange devices have enabled extensive communication among decision makers in the last decades. This emerging communication pattern creates access to a wealth of knowledge, the opportunity to seek assistance anywhere, and channels to obtain real-time feedback. As a result, transportation activity patterns are reshaped; extensive mutual selection and joint decision making can be observed widely, and neglecting this trend would result in insufficient understanding of related issues.

The emerging communication pattern is supported by networks comprised of a collection of decision makers. This section will first review the typical problems and underlying economic theory about collaboration and joint decision making. Based on characteristics of collaboration and research objectives, agent interactions have been analyzed by various quantitative methodologies, which will be revisited next. These quantitative methodologies can be attributed to agent- and optimization-based models. However, these methodologies have to presume certain behavior rules. On the other hand, econometric modeling extracts information from collected data without presumed rules. Although a few studies have accomplished efforts in these models, the exploration has not been comprehensive. The main contribution of this dissertation is to develop an innovative econometric model to fill the void, providing important insights into collaborative transportation activities. Note that the reviews of typical problems and methodologies are not mutually exclusive, but have different focal points. The literature review ends up with the review of studies related to the two empirical applications of this dissertation: the airline-airport collaboration and the freight carrier-customer collaboration.

2.1 Typical Problems Related to Joint Response

Collaborative activities are usually conducted through a collection of decision makers who are seeking common goals and benefits. The relationship of decision makers can be very complex in terms of relative power, complementarity of decision makers' functions, collaborative capacity, and many other aspects that shape collaborative activities. A comprehensive and ideal analytic framework should include all aspects as the research target. However, doing such would likely result in a very complicated and infeasible reasoning process. Therefore, existing studies usually simplify the analytic framework by analyzing a certain type of collaboration.

Therefore, based on the type of collection, literature has put great efforts in solving problems according to the specific context. If the decision makers come from a two-side market, such as a marriage market, the marriage problem focuses on disentangling the matching process between agents of two sides. On the other hand, if the collaborators belong to one single side, the group decision making problem aims at understanding the negotiation procedures among different decision makers. Furthermore, inter-organizational relations problems can analyze how organizations interact with each other and conduct collaborative economic activities. In recent years, an emerging research field, sharing economy, intends to account for the peer-to-peer-based sharing of goods, services, and capacity.

2.1.1 Classic Matching Problem

One of the most classic matching problems is the marriage problem where decision makers look for the best counterparty in a two-side market. A classic marriage problem aims at finding a stable matching between two equally-sized sets of agents given a ranking of preferences for each agent. A matching is "stable" when a man and a woman are both engaged, but not to each other, and upon the completion of the matching, it is not possible for them to prefer each other over their current partners. In 1962, Gale and Shapley (1962) published the paper "College admissions and the stability of marriage". They tried to find whether there is a stable way to match men and women so that no unmatched pairs are left. The Gale-Shapley algorithm proved that it is always possible to solve the stable matching problem and make all marriages stable for any equal number of men and women. In specific, their algorithm formulates a number of actions to propose and accept/reject until everyone is engaged. Their work mathematically proved that there cannot be a man and a woman both unengaged, as the unengaged man would eventually propose to the woman at some point. Although the idea and algorithm are straightforward, Gale and Shapley's work was a fundamental study that started the 60 years' matching market studies in finding solutions to real-world matching problems. Their work was recently honored by the Nobel Prize in Economic Sciences in 2012 (The Nobel Foundation, 2012).

The variants of Gale-Shapley algorithms soon served as the methodologies to solve a number of matching problems, such as matching doctors and hospitals (Roth, 1984), matching students and high-schools (Abdulkadiroğlu, Pathak, & Roth, 2005), and matching kidneys and patients (Shapley & Scarf, 1974). The idea behind the algorithms is to solve problems that violate traditional economic theory where prices adjust the supply and demand. When prices do not function well and rational people know their best interests, the matching algorithms could allocate resources in an efficient way. This type of algorithms and the concept of stability are usually perceived as important components in cooperative game theory, an abstract area of mathematical economics, which aims at determining the best way to cooperatively choose an allocation between rational individuals.

In the transportation field, the marriage problem is not commonly formulated because matching of decision makers has not been given sufficient importance. The limited number of studies on ordinal transportation problems attempt to seek explanations on agent matching in the freight supply chain (Baïou & Balinski, 2002; Ostrovsky, 2008). In recent years, several taxi scheduling papers have adopted the idea of the marriage problem to efficiently match taxi drivers and riders given the fast developing real-time information exchange technology (Bai, Li, & Kendall, 2013; Wang, Agatz, & Erera, 2014; Thaithatkul, Seo, Kusakabe, & Asakura, 2015). Studies on parking (Ayala, Wolfson, Xu, DasGupta, & Lin, 2012; He, Yin, Chen, & Zhou, 2015) have also borrowed the idea of stable marriage problem to formulate parking competition games. In fact, a lot of other transportation-related activities involve the matching process, such as the empirical application of this dissertation – airline-airport matching. In addition, the idea can be also employed in the public-private partnership, peer-to-peer travel enabled by tourist-local guide partnership, and co-housing problems.

The marriage problem serves as a foundation for disentangling the intricate matching network in the proposed joint response model. This utility-based process to propose and accept/reject is not the main focus, but the proposed model considers its converse process: based on the observed accept/reject results, utility between matched and unmatched pairs can be inferred. In other words, the proposed model admits that individuals have already conducted rational choices on partners and the market has reached stability. Econometric analysis would be conducted on the basis of understanding relative utility over all pairs of decision makers.

2.1.2 Group Decision Making

If researchers want to understand the decision outcome of matched agents, the group decision making problem is a prototype to understand how the joint decision is reached. Group decision making is a process faced by multiple decision makers to collectively determine the best alternative that is available to them. Unlike individual decision making, group decision making does not attribute to any single individual but the entire group. A group decision is usually reached by a voting or a consensus process that may involve discussions and compromise. As a result, the decision outcome of a group is usually different from the outcome of an individual. Although there is much debate as to whether a group decision is better than an individual decision, group decisions present superiorities over individual decision making in the following aspects. Groups can represent a diverse set of perspectives so that fair opinions are likely reached. In addition, group decisions are believed to reach better results in important decision making. Due to these advantages, group decision making plays an important role in the history of human civilization, such as in voting, legislature, and jury trials. It can be also observed in businesses (e.g., sales teams), education (e.g., school boards), and many other fields that involve collaborative decision makers.

Methodologies of group decision making can be seen by two aspects: qualitative models and quantitative models. Early works of qualitative models focused on how individual preference affects the group's choice (Sniezek & Henry 1990; Tindale, Kameda, & Hinsz, 2003). Recent works started to analyze how information is processed through the group members (Hinsz, Tindale, & Vollrath, 1997). The objectives of both studies were to understand factors that influence group decision making and assure a successful decision making process. Vroom (2003) developed a normative model of group decision making, which consists of five processes: decide, consult (individually), consult (group), facilitate, and delegate. As for quantitative analyses, group decision models are extensions of utility theory in classic economic theory. The key concern is how to properly aggregate individual utility to a group level. Some utilitarians argued that utility functions of individuals are comparable so that understanding group decision making can be measured at the maximum of summed utility. Opponents, represented by Sen (1970), claimed that only partial comparability of utility is possible, which is the key problem in the social choice theory, a theoretical framework for analysis of combining individual preference or welfare (Arrow, 2012).

In the transportation field, group decision making is mostly considered in travel pattern analyses within household (Davis, 1976; Bhat & Pendyala, 2005; Timmermans & Zhang 2009; Zhang, Kuwano, Lee, & Fujiwara, 2009) and tourist groups (Thornton, Shaw, & Williams, 1997). These studies qualitatively and quantitatively analyze the interactions among decision makers, and model the decision making process with the consideration of interactions. However, these studies assume that the formation of groups is pre-defined and not affected by the joint decision making process. This assumption is valid in some situations, such as household travel, but invalid in other situations, such as certain group travel cases. For example, a participant may quit a group if he is not satisfied with the travel destination. This dissertation considers the matching and joint decision making as two simultaneous processes, yielding consistent understanding of frequently changed matching structures in real-world problems.

2.1.3 Inter-Organizational Relations

A group decision is not effective if neglecting its environment. In this case, the inter-group/organization relations need to be considered. Based on the premise that organizational collaboration leads to a more coordinated way to address complex problems, inter-organizational relations theory focuses on how organizations work together. The necessity of developing this theory is that organizations are embedded in an environment of other organizations, which features a complex of norms and values (Evan, 1965). As a consequence, an organization is not able to act freely to maximize its own benefit, but considers the interactions with other organizations. The interaction can be very complicated in several aspects. The environment may have concentrations on a certain organization which possesses key resources, resulting in unbalanced market power. The members of an organization may overlap with other organizations, leading to coalition between them. Two organizations can also perform as input and output organization where business strategies play a crucial role in their interaction. Due to the complexity of the real-world problems, literature has to simplify the problem by focusing on only some aspects.

An important simplification to this dissertation in analyzing inter-organizational relations is the input-output model. Pioneered by Wassily Leontief (Leontief, 1936), input-output models are used to analyze the interdependencies among industries. The basic form of an input-output model consists of a system of equations in which each one formulates the distribution of an industry's product over the market (Miller & Blair, 2009). The input-output model has been employed in many empirical analyses related to the transportation field. For example, Robison and Miller (1988) used input-output models to study the timber economy of the West Central Idaho Highlands and found that input-output models were efficient techniques in cross-region trade analyses. Hewings et al (2001) employed an input-output framework to investigate the interdependence of inner-city communities and suburbs of the Chicago metropolitan area.

The proposed model in this dissertation can solve a variety of joint decision problems in a two-side market, in which the two sides are usually the upstream providers and downstream consumers. Different from classic input-output models, the proposed model mainly focuses on the input and output within one industry rather than cross-industry interactions. In addition, the proposed model is able to capture the matching process for upstream and downstream decision makers.

2.1.4 Sharing Economy

The marriage problem characterizes the individual's choice on its partner, leading to a group. Joint decisions achieved by the collection of group members can be understood as a group decision making problem. The inter-organizational relations problem further features that group decisions have to consider that the decision makers' environment and partner selection can also occur at the organizational level. Essentially, these three problems relate to different levels of reaching a joint response.

Joint responses are observed widely in society, especially in the past five years when information and communication technology are explosively emerging. Two representative examples that sweep the creative industry in this era are discussed below.

Airbnb, an online housing search platform that helps people to find and rent out places, has served more than ten million guests within the five years since its launch (Lawler, 2013). The successful business builds on the business model where it connects hosts and travelers without owning any rooms itself. Such a model disrupts traditional hotel industry by creating a new source of supply. Unlike the traditional hotel industry, Airbnb scales by increasing the number of hosts and travelers and matching them with each other (Lawler, 2012).

Uber operates the Uber mobile app to connect riders with drivers. Founded in 2009, Uber has ranked in the top 50 most powerful companies in the U.S. and is estimated to be worth $62.5B (Newcomer, 2015). This on-demand taxi service matches travel demand with travel supply in an efficient way. It reduces the cost of searching and matching between riders and drivers. As a result, congestion is relieved because unnecessary driving around is reduced; less built-up neighborhoods can easily take taxi service; and drunk driving rate is reduced (Lyft Uber Newsletter, 2016).

Similar business models related to transportation can be also found in ToursByLocal, Zipcar, and home exchange. These activities feature a two-side market where users of one side match with users of the other side. These businesses provide platforms to facilitate matching processes and enable the optimization of resource use through redistribution, sharing, and reuse of excess capacity in goods and services. This emerging peer-to-peer collaboration phenomenon is referred as sharing economy, which has drawn increasing attention since its first appearance in the early 2000s. Its synonyms "collaborative consumption" was named one of ten ideas that will change the world (Walsh, 2011).

The development of sharing economy is driven by a number of factors. Sundararajan (2014) summarizes the key drivers as the consumerization of digital technologies, the emergence of digital institutions, urbanization and globalization, and ecological and resource considerations. Digital technologies create the possibility of peer-to-peer business. Digital institutions facilitate economic exchange, ensuring the smooth interaction between peers. Coupled with changes in lifestyle, sharing economy is taking and will constitute a significant segment of the economy in the coming years. From the perspective of transportation economy, owning unused goods and service is costly, resulting in the imperative necessity of renting them to others in need. The use of excessive transportation supply enables the fast developing of peer-to-peer economy in the transportation field.

Although sharing economy is believed to bring convenience and lower costs, and to improve economic development, the research community faces challenges that prevent it from fully understanding its economic effects. One obstacle is at the macro-level: its economic contribution is hard to measure because many transactions occur between individuals and involve re-utilization. As a result, sharing economy's effect on traditional industry and the entire economy has not been concluded. Some studies have attempted to explore the maze of sharing economy. For example, Zervas et al. (2015) use empirical data to analyze Airbnb's economic effect on the traditional hotel industry. Another concern is at the micro-level: a matching market may have imbalance in supply and demand, which prevents the market from fully functioning. The supply and demand relationship can be twofold: the first concern is about the collaborator (e.g., the market of selecting partners) and the second concern is the goods or service (e.g., the market of purchasing goods and service). For example, Airbnb at the early stage had to use special strategies (e.g., hire photographers to take nice room pictures) to increase the number of hosts and available housing. Another concern at the micro-level is the understanding of joint decision making because decisions on mutual selection and implementing travel activities are simultaneous: If joint decision making cannot be reached, the group would also break up. In the existing literature, problems at the micro-level are mostly discussed without the support of empirical studies (Belk, 2014; Cohen & Kietzmann, 2014; Hamari, Sjöklint, & Ukkonen, 2015). The reasons might be that (1) empirical data is hard to obtain because the data related to companies' operation is confidential, so that companies would not release the data to the public, and (2) no sound quantitative methodologies can deal with this emerging economic phenomenon. In light of these challenges, the proposed model aims at developing innovative methodologies to enrich the understanding of sharing economy.

2.2 Methodologies Related to Joint Response

Given the challenges in understanding joint response, a variety of quantitative methods have been proposed to analyze collaborative behavior. The most popular methodologies are agent-based modeling and cooperative game theory, optimization-based approaches, and econometric modeling. Agent-based modeling and cooperative game theory focus on simulating each decision maker's behavior and how each individual interacts with others. Optimization-based approaches attempt to find optimal solutions to the benefit of the matched collaborators. These two methods presume certain behavior rules and researchers understand each decision maker's behavior based on these rules. On the other hand, econometric modeling does not presume behavior rules. It extracts important information directly from the collected data.

2.2.1 Agent-based Modeling and Cooperative Game Theory

When there are interactions of multiple autonomous agents, agent-based modeling is able to understand properties of complex social systems through simulation (Axelrod, 1997). Each agent in the model is assumed to behave on the basis of assessing its own situation and its behavior can change frequently when agents need to adjust to the changing environment. A simplest agent-based model defines simple rules for agent's behavior and the investigated problem is considered only during a fixed time period with a limited number of agents. However, such a simulation may be still complicated because agents would go through adaptation processes. Agent-based models are particularly useful when agent interactions are heterogeneous and agent behavior is not assumed rational. Through simulating each agent's behavior to obtain measurement of the entire system, agent-based modeling can provide important insights into the real-world problem. Agent-based modeling does not have a mature set of standard procedures for model development (Macal & North, 2005). Definitions of agents, interaction relationship, and behavior rules are mostly case-specific. In the transportation-related studies, agent-based simulation is used in household travel behavior analysis to capture the interaction among household members (Ronald, Arentze, & Timmermans, 2012). It is also used to simulate the interaction of supply chain participants (Swaminathan, Smith, & Sadeh, 1998).

A related methodology of agent-based modeling is game theory, which studies the conflict and cooperation between rational decision makers. Similar to agent-based modeling, game theory also focuses on understanding the behavior of individual decision makers considering interaction with other decision makers. However, the research objectives of the two methods are different. Game theory aims at exploring axioms that can deduce behavior of decision makers, which advises rational decision makers in similar problems. On the other hand, agent-based modeling does not require decision makers to be rational or prove any theorems. It attempts to find useful patterns inductively from simulation.

Game theory has been extensively studied in the second half of the last century. It helps to explain problems in transportation, economics, political science, psychology, and many other fields. Based on the nature of problems of interest, a variety of game types are modeled. Among them, the cooperative game theory is mostly related to the topic of this dissertation, which explores players' behavior when they form binding commitments. A cooperative game focuses on coalition of individual players, competition among coalitions, and the consensus decision making process. A key assumption in cooperative games is that players could transfer utility with other players in the same coalition. In this dissertation, the partners in a two-side market can also be treated as a coalition, leading to pairwise utility to characterize the behavior of coalition. Within each pair, utility of one side is transferrable to the other side. The assumption of using pairwise utility in this dissertation borrows the idea of transferrable utility in cooperative games.

Cooperative game theory is widely used in the transportation field. For example, supply chain management uses cooperative games to understand the interaction among agents (Li, Huang, Zhu, & Chau, 2002; Cachon & Lariviere 2005; Cruijssen, Dullaert, & Fleuren, 2007; Krajewska, Kopfer, Laporte, Ropke, & Zaccour, 2008; Esmaeili, Aryanezhad, & Zeephongsekul, 2009). Roadway network planners use cooperative games to coordinate information providers, drivers, and traffic authorities (Adler & Blue, 2002). Driving behavior can be also investigated by cooperative games, such as the investigation of merging vehicles (Kita, 1999). The revenue sharing of airlines and airports is also analyzed by game theory (Saraswati & Hanaoka, 2014; Yang, Zhang, & Fu, 2015).

2.2.2 Optimization-Based Approach

Agent-based modeling and cooperative game theory are interested in analysis of an individual's behavior considering the interaction with other agents. Results would serve for measuring the economic impacts and policy-making. Another set of methodologies that helps to understand joint response is operation research, which has a strong computational orientation and intends to assist with decision making. Generally, operation research methodologies aim at finding optimal solutions to complex decision-making problems with subject to certain constraints. Typical models maximize profits/performance or minimize loss/risk. If the optimization is modeled for an individual or an entity, the model is called local optimization, which contrasts to global optimizations that optimize objectives for all participants in a problem.

Global optimization is particularly useful in understanding the supply chain management where multiple agents coordinate to optimize the benefits of all participants and alignment of decisions between them. In a typical objective function, the total cost of supply chain is minimized rather than only one agent's cost (Kheljani, Ghodsypour, & O'Brien, 2009; Kamali, Ghomi, & Jolai, 2011). The objective function can be also the use of capacity, collaborative benefits, security (Meixell & Norbis, 2012), punctual delivery and other measurements related to supply chain management (Yao, 2013). As global optimization often involves multi-objectives, integer numbers, and multi-level problems, solving the global optimization is not straightforward. As a result, cutting-edge research is to propose feasible and efficient methods (Aliabadi, Kaazemi, & Pourghannad, 2013; Paksoy, Özceylan, & Weber, 2013).

This dissertation does not analyze agent collaboration by finding optimal solutions, but by analyzing the causality between influential factors on joint responses. Therefore, the comparison of the proposed model and optimization-based approaches is similar to the comparison of operation research and regressions in general: the two methodologies look at a problem from different perspectives and have different reasoning processes.

2.2.3 Econometric Modeling

Unlike the other two methodologies, regression-based models extract important information from empirical data to explain the relationship between factors in interest. In transportation, regression models use collected data to explain and forecast travel patterns. These models are widely used in all steps of the traditional sequential four-step travel demand model (Manheim, 1979). They are also used in analyzing service performance, safety, land use, and a lot of other important topics. Standard regression analysis treats one individual or entity as the sole decision maker, and various advanced variants have been developed to further capture panel data, temporal correlation (Zhang & Wang, 2015), spatial effects (Zhang & Wang 2015; Zhang & Wang 2014), and individual heterogeneity (Zhang, Magalhães, & Wang, 2014). However, travel patterns produced by multiple decision makers have not been given sufficient focus. Although studies have pointed out the importance of interaction among multiple decision makers (Holguín-Veras, Aros-Vera, & Browne, 2015), sound quantitative methodologies are still highly demanded for the research community. Most regression models involving multiple decision makers analyze intra-household interactions (Srinivasan & Bhat, 2005; Zhang, Kuwano, Lee, & Fujiwara, 2009). This series of studies assume the connection among decision makers is pre-determined and not related to the final decision outcome. As for the explanation of why decision makers decide on collaborating with each other, only a few matching models partly respond to the concern.

Therefore, this section of literature review will start to introduce these matching models, which relates to the main contribution of this dissertation. The proposed model investigates a many-to-many matching network (compared to the one-to-many matching network in the existing literature), ordinal/multinomial discrete outcome (compared to the continuous/binary outcome), and a flexible variance-covariance matrix in the sample selection process (compared to a restricted variance-covariance matrix). In addition, the review of spatial interaction will be followed because the proposed model is able to investigate spatial interaction at the disaggregate level, filling the void of lacking methodologies of disaggregate spatial interaction analysis. Finally, sample selection models are discussed in order to highlight the contribution of the proposed model, which uses the matching result to determine subsamples. Collectively, the proposed method extends the existing methodologies from multiple perspectives, highlighting the significance of this dissertation.

2.2.3.1 Matching Model

Methodologies analyzing the formation of collaborative relationship have a relatively short history. In 2010, Dale Mortensen earned the Nobel Prize in Economics for the analysis of markets with search frictions, which explicitly explain the process of partner selection. Matching model frameworks can explain phenomena that are raised by multiple decision makers. When the matching relationship is observed, these models can analyze the effect of influential factors on forming the observed matching. Matching data include which firms do business with which firms, which men are married to which women, and which players are teammates with which players, among other data involving collaboration. The basic economic idea is that one individual would like to match with the most attractive partners, leading to the highest benefits for the individual. Econometricians seek influential factors that determine the observed matching and estimate the parameters of these factors.

An important matching study is Sorensen (2007), which uses a two-sided matching model to explain firms' IPO with the bank-firm matching. It first uses a latent variable equation to explain the bank-firm partner selection and then uses a binary outcome model to formulate firms' IPO. The first equation considers matching utility of all possible pairs while the second equation only considers the IPO of matched pairs. A Bayesian MCMC approach is employed to estimate parameters of factors in determining all parameters in the two equations. This modeling framework models each pair's behavior to analyze the selection of partners. Such a method is a fundamental work for this paper, which extends the matching structure from a one-to-many (e.g., firms can only get invested from one bank) to a many-to-many (e.g., each supplier can trade with multiple customers, and each customer can trade with multiple shippers) network. The outcome of joint decisions can be ordinal and multinomial, compared with continuous/binary outcomes. A flexible variance-covariance matrix is also specified to understand the sample selection process.

Similar econometric matching models have been discussed in a limited number of empirical studies. Chen (2013) specifies the utility equations for each side of the partner to analyze the premium of bank loans. This study is an extension of Sorensen (2007) in which the utility of paired partners is investigated instead of utility of decision makers, respectively. However, this model may suffer from identification problems if extended to discrete outcomes. To the author's best knowledge, no other studies have conducted the research in a similar way. Other studies have analyzed the matching process from the market's perspectives, using different estimation methods, or without considering the mutual selection process. For example, Choo and Siow (2006) investigated the stable matching relationship from the market's perspective rather than from each decision maker's perspective. Similar marriage analyses can be also found in Siow (2008). Hitsch et al. (2010) also analyze a marriage dataset, but the methodology does not consider the mutual preference by sorting pairwise utility. Fox (2008) and Levine (2007) use maximum score estimators (e.g., a non-parametric model) to identify parameters in the matching model.

2.2.3.2 Spatial Interaction

Another important issue related to regression models is spatial interaction because decision makers locating at different places often present certain spatial relations. For a long time, the spatial interaction analysis mainly relies on aggregate models, such as input-output models (Leontief, 1941; Isard, 1956) and gravity models (Tinbergen, 1962). These models assume that decision makers can be represented by homogeneous geographic regions, usually census units configured based on geography, economics, and administrative divisions. For example, trade flow is usually conceptualized by characteristics of the origin and destination counties, rather than disaggregate corporations and individuals who ship and receive the goods. Such an aggregation allows for lower data requirements and computational burden, but was unable to capture the heterogeneity of decision makers.

The most frequently used method in analyzing travel demand with the consideration of spatial relationship is the gravity model. Gravity models characterize that the travel demand between large economics is stronger than between small ones, and nearby economics attract each other more than faraway ones. Recent research of gravity models mainly focus on resistance terms (Rose and Van Wincoop 2001; Baier and Bergstrand 2009), zero trade flows (Silva & Tenreyro 2006; Helpman, Melitz, & Rubinstein, 2007), and distance measurements (Limao & Venables 2001; Disdier & Head 2008). An important factor in gravity models, distance is specified by different methods. The most commonly used method is the distance between the centers of the investigated regions. Based on the characteristics of regions, centers are usually capitals, largest cities, or the centroids. Adjacency is also considered in some literature, which is due to the consideration of freight costs and political costs. Other subtle factors, such as trade cost, market access, economic geography, and language similarity are also perceived to capture the concept of distance in gravity models.

However, these aggregate analyses assume that travelers can be represented by homogeneous geographic regions, usually census analysis units based on geography, economics, and administrative divisions. For example, freight flow is usually conceptualized by characteristics of the origin and destination counties, rather than disaggregate corporations and individuals who actually ship and receive the cargos. Such aggregation allows lower data requirements and computational burden, but is unable to capture the heterogeneity of individual travelers.

The development of discrete outcome models (McFadden, 1972) enables studies of travel demand at the disaggregate level. However, constrained by the behavioral framework, these models have to assume that travel decisions are made by one individual. For freight travel demand, either the supplier or the customer, but not both, determines the delivery activity. The characteristics of the customers (or suppliers) are sometimes used as exogenous variables to help explain the supplier's (or customers') behavior. Later, group decision models, mainly focusing on intra-household collaboration, were investigated to recognize the fact that multiple agents may jointly make decisions. Srinivasan and Bhat (2005) investigated intra-household activity travel patterns by examining interactions among household members. Zhang et al (2009) investigated household discrete choice behavior considering heterogeneous group decision making mechanisms. However, the matching relationship of household members is not impacted by the joint decision making process.

One key limitation of the existing disaggregate models is the treatment of decision makers. Spatial activities often involve at least decision makers located at different places (e.g., shipper and receiver for goods flow, and worker and employer for commuting flow). Focusing on one side or treating both sides as one group implies that (1) the spatial relationship between the decision makers is not explicitly recognized, and (2) the bonding between the two sides is considered predetermined and will not be impacted by the decisions they make for spatial activities.

Such assumptions may be acceptable for spatial activities generated by decision makers in long-term binding contracts, such as workers' commuting trips and bulk deliveries between large manufacturers and stores. However, a growingly large share of spatial activities seems to be generated by temporarily paired-up decision makers. For these spatial activities, the matching of decision makers is endogenous to the decisions they make. Moreover, the spatial relationship between the decision makers plays a key role in the matching process.

2.2.3.3 Sample Selection Model

The proposed model in this dissertation models the formation of collaboration and the joint response of matched decision makers as a simultaneous process. The basic idea is to specify two equations with each equation capturing one process. An important issue of the two equations is that the number of samples in each equation is different from each other. The first equation models all potential pairs but the second equation models only the matched pairs. Such a type of cross-equation relationship is analogous to a sample selection process. To analyze such a process, sample selection models can provide important insights. Standard sample selection models use binary response select subsamples, and the objective is to correct the bias resulted from non-random sampling (Heckman, 1979; Wooldridge, 2010; Goldstein, 2011). The selection equation (i.e., the first equation) is a binary outcome model constituting all samples. The outcome equation is a continuous model with only samples that have affirmative answers in the binary outcome model. In the transportation field, Rashidi et al. (2012) uses a sample selection model to correct the bias resulting from selecting samples from a particular type of neighborhood. Anderson et al. (2012) employs a sample selection model to identify the occurrence of railway track renewal and analyzes its costs in Sweden. Vance and Iovanna (2007) investigates the determinants of automobile travel demand by considering gender by a sample selection model. The sample selection bias is tested by the significance of correlation between the selection equation and the outcome equation. If the two equations are found independent, a sample selection model may reduce to a two-part model.

The proposed models of this dissertation can be seen as an extension of the standard sample selection model: the binary outcome model is replaced by a matching equation and the continuous model is extended to ordinal and multinomial outcome models. The groundwork of this dissertation, Sorensen (2007), also specified his model in a similar way:

a matching equation and a binary outcome equation. This dissertation further investigates the many-to-many matching structure and ordinal/multinomial outcomes.

2.3 Literature Related to Applications

The last section of literature review discusses important works related to the two applications in this dissertation.

2.3.1 Airline-Airport Collaboration

Airline-airport vertical relationship has drawn increasing attention from the research community. With the commercialization, privatization, and liberalization, both parties start to treat each other as business partners and have formed various collaborative relationships. As a result of collaboration, traditional airline companies are able to implement their hub-and-spoke network operations, leading to a better profit earning strategy. Low-cost airlines are free in choosing airports as their base to improve their market shares. The airline-airport collaboration has brought positive outcomes: high profits, low airfare, and local economic development. It also raises concerns about anti-competitive consequences (e.g., an airline's dominance is strengthened at an airport to gain competitive advantages over other airlines), poor service performance of non-signatory airlines (e.g., airlines who do not use the airport as a hub), and moral hazard (Hihara, 2011).

The types of airline-airport collaboration can be characterized as follows. Fu et al. (2011) summarize the relationships as signatory airlines of airports, airline ownership or control of airport facilities, long-term use contracts, airport issuance of revenue bonds to airlines, revenue sharing between airports and airlines, and other agreements. Albers et al. (Albers, Koch, & Ruff, 2005) discuss the relationship in different countries. In the U.S., the relationship can be that the airport serves as a landlord and a coordinator of services. The airline builds their terminals and facilities. In Spain, one central public airport company owns and develops all the airports of the country and airlines are just customers. In France and the U.K., airport companies can be either public or private and airlines are also just customers.

The research in airline-airport collaboration is not extensive mainly because price discrimination is not allowed by International Air Transport Association (IATA), leading to a restricted market. Some important studies are discussed below.

Barbot (2009) uses a three-stage game to understand airport and airline competition. A set of incentives for vertical collaboration is discussed and results find that collaboration exists when airports and airlines have different market sizes, and when secondary airports and low-cost airlines compete with main airports and full service airlines.

D'Alfonso and Nastasi (2012) develop a facility-rivalry game to analyze vertical contracts between airports and airlines in the context of three types of agreements. This study finds the Nash equilibrium to analyze incentives for vertical contracts and the effects on welfare, consumer surplus, and pro-competitiveness.

Zhang et al. (2010) investigate the revenue sharing between airports and airlines using a game theory analysis. Depending on different economic effects between each party (e.g., complements, independent, or substitutes), the sharing structure is significantly different.

To the author's best knowledge, the above-discussed studies are the only ones using quantitative methods to analyze collaboration between multiple airports and multiple airlines. Therefore, the application of this dissertation will enrich the airline-airport collaboration literature by using econometric models to analyze the formation of collaboration.

2.3.2 Freight Agent Collaboration

The freight supplier-customer collaboration is used as an example throughout the dissertation because collaboration is typical behavior for freight transportation. Freight systems transport supplies needed for people's daily life, generating tremendous benefits as well as externalities of environment impacts. MAP-21 (Moving Ahead for Progress in the 21st Century) acknowledges the importance of freight research to support policy making (U.S. Department of Transportation, 2013) and calls for the design of effective freight policies to reduce congestion, mitigate pollution, and improve supply chain efficiency.

The importance of collaboration in freight transportation has been discussed in the literature although sound quantitative methodologies are still highly demanded. A series of freight delivery time studies (Holguín-Veras, Silas, Polimeni, & Cruz, 2008; Holguín-Veras, Xu, De Jong, & Maurer, 2011; Holguín-Veras et al., 2015) has raised concerns over freight agents' interactions. Freight carriers prefer the nighttime due to smooth traffic and lower costs, but product receivers prefer the daytime because no additional labor is needed to receive cargos during business hours. The delivery time determination can be treated reasonably as joint decision making between carriers and customers. In addition, Holguin-Veras et al. (2015) argues that a lot of freight activities are also the result of the freight agents' interactions, and that these interactions determine the supply chain's response to freight policies. For example, delivery rates, sizes, and frequency are impacted jointly by suppliers, carriers, and receivers. Disregarding interactions among agents may prevent the research community from fully understanding the decision mechanism, leading to misleading assessments of policy effects on each individual decision maker and consequently, poor predictive power. Given the gap between the observed collaborative activities and the existing analytic framework, the proposed models will fill the void by behavioral-consistently formulating collaboration behavior across freight agents.

3. Model Specification

The main objective of this dissertation is to develop a joint response model to analyze new trends of collaboration in transportation. From different views of econometric modeling, the proposed model contributes to the existing literature of limited dependent variable models and sample selection models. The limited dependent variable is the pairwise utility in the matching equation, aiming to disentangle the many-to-many matching network between agents in a two-side market. The sample selection process is captured using the matching results and a flexible variance-covariance matrix.

3.1 Matching Equation

The matching equation characterizes the mutual selection process between agents in a two-side market (e.g., suppliers and customers in a freight market). This equation is specified based on the following assumptions: (1) Each agent is assumed to have full information of all agents on the other side and intends to look for the best partner from the other side; (2) Each agent could match with one or multiple partners of the other side; (3) Each potential pair has a pairwise utility, which values the preference of mutual selection. And (4) The matching relationship data is stable: no agent prefers to deviate from current pairs and form a new pair with another agent.

Let $i\left(i=1...I\right)$ denote a set of suppliers and $j\left(j=1...J\right)$ denote a set of customers in a two-side market. The number of possible pairs in the market is $I \times J$. Let N_0 and N_1 denote the collections of unmatched and matched pairs, respectively. Thus, the collection of supplier i's unmatched pairs is denoted as $N_0\left(i\right)$ and the collection of supplier i's matched pairs is denoted as $N_1\left(i\right)$. Note that a certain matched pair between supplier i and customer j can be stated as either $j \in N_1\left(i\right)$ or $i \in N_1\left(j\right)$. Let u_{ij} denote the matching utility of pair ij. Assume the utility of all possible pairs are distinct.

As the matching relationship is assumed stable, a set of inequality conditions can be inferred to characterize the relative magnitude of pairwise utility u_{ij}. The inequality conditions for each unmatched pair u_{ij} is

$$u_{ij} < \overline{u_{ij}} = \max\left[\min_{j'\in N_1(i)} u_{ij'}, \min_{i'\in N_1(j)} u_{i'j}\right] \tag{3.1}$$

The term $\overline{u_{ij}}$ is the opportunity cost for supplier i or customer j to deviate from their existing pairs and form a new match together with each other. An unmatched pair remains unmatched in a stable market when at least one side is unwilling in the counterparty. For supplier *i*'s side, the current matched pairs should be better than the proposed pair *i-j*. Even the worst current matched pair of *i* is better than the proposed pair. $\min_{j'\in N_1(i)} u_{ij'}$ is the utility of the worst current matched pair of *i* and serves as a threshold. As long as the utility of the proposed pair is smaller than this threshold, supplier *i* is unwilling to matched with customer *j*. As this is a mutual selection process, the same process can be found on the customer *j*'s side: $\min_{i'\in N_1(j)} u_{i'j}$ is the utility of the worst current matched pair of j and serves as a threshold. As long as the utility of the proposed pair is smaller than this threshold, customer *j* is unwilling to matched with supplier *i*.

If any side is unwilling to match with the counterparty, the pair cannot be matched. Hence, if the utility of the proposed pair is smaller than any of the two thresholds, the pair is unmatched, resulting in a maximum for the outermost parenthesis.

A matched pair u_{ij} is constrained by the following conditions:

$$u_{ij} > \underline{u_{ij}} = \max\left[\max_{j'\in S(i)} u_{ij'}, \max_{i'\in S(j)} u_{i'j}\right] \tag{3.2}$$

where $S(i) = \left\{ j \in J : u_{ij} > \min_{i'\in N_1(j)} u_{i'j} \right\}$ and $S(j) = \left\{ i \in I : u_{ij} > \min_{j'\in N_1(i)} u_{ij'} \right\}$.

The term $\underline{u_{ij}}$ is the opportunity cost for supplier i and customer j to stay with their existing pairs. A matched pair remains matched in a stable market when both sides are unwilling to match with any feasible deviations. The feasible deviations are denoted by $S(i)$ and $S(j)$, respectively. For the supplier i's side, the feasible deviation is a collection of j who would accept i's propose surely if i proposes to them. If the proposed deviation is better than any of the current matched pair of the proposed deviated customer

j. $\min_{i' \in N_1(j)} u_{i'j}$ is the utility of the worst current pair and serves as a threshold. As long as the utility of the proposed deviation is greater than this threshold, the corresponding j is a feasible deviation of i. The feasible deviations of customer j can be derived using the same method.

Both sides are unwilling to match with any feasible deviation, indicating that the utility of the matched pair should be better than all feasible deviations. Hence, the maximum of the maximum is used to capture such a relationship.

The pairwise utility can be attributed to a series of explanatory variables, which can be expressed as a regression equation (e.g., the matching equation in the proposed models)

$$u_{ij} = \alpha w + \eta_{ij} \tag{3.3}$$

where

$$\alpha = \begin{bmatrix} \alpha_i & \alpha_j & \alpha_{ij} \end{bmatrix},\ w = \begin{bmatrix} w_i \\ w_j \\ w_{ij} \end{bmatrix}.$$

The explanatory variable w includes factors of supplier i (e.g., denoted as w_i), customer j (e.g., denoted as w_j), and their joint factors (e.g., denoted as w_{ij}). The α consists of the corresponding parameters to be estimated. The error term η_{ij} contains unobserved effects determining the pairwise utility and is assumed to follow a normal distribution. This error term can be also decomposed to include supplier side errors, customer side errors, and joint error. However, such decomposition leads to identification concerns and is left for future works.

As pairwise utility is a constrained variable, parameters in the matching equation can be identified. This type of models and estimation techniques is similar to limited dependent variable models, such as logit, probit, and Tobit models. This is the reason why the proposed model contributes to the literature of limited dependent variable models.

3.2 Joint Decision Making Equation

Matched agents jointly make decisions, which can be continuous or discrete outcomes.

3.2.1 Continuous Outcome

The continuous outcome can be modeled by a standard linear regression equation. The continuous joint decision outcome y_{ij} is expressed as

$$y_{ij} = \beta x + \varepsilon_{ij} \tag{3.4}$$

where

$$\beta = \begin{bmatrix} \beta_i & \beta_j & \beta_{ij} \end{bmatrix},\ x = \begin{bmatrix} x_i \\ x_j \\ x_{ij} \end{bmatrix}.$$

Similar to the matching equation, the term x contains influential factors of supplier i, supplier j, and their the joint factors. The β contains the corresponding parameters to be estimated. The error term ε_{ij} is assumed to follow a normal distribution.

3.2.2 Ordinal Outcome

The ordinal outcome y_{ij}, such as delivery frequency, can be analyzed using an ordered probit model. The observed outcome can be modeled as

$$y_{ij}^* = \beta x + \varepsilon_{ij}$$

$$y_{ij} = C \text{ if } \mu_{C-1} \le y_{ij}^* < \mu_C \tag{3.5}$$

where definitions of variables are similar to those in the continuous case. The difference is the treatment of outcome. Ordered probit models use C to denote the observed ordinal outcome and threshold μ_C to divide a continuous latent variable y_{ij}^*.

3.2.3 Multinomial Outcome

The multinomial outcome y_{ij}, such as mode/route/time choices, can be analyzed using a multinomial probit model. The observed outcome can be modeled as

$$y_{ij,p}^* = \beta_p x_p + \varepsilon_{ij,p}$$

$$y_{ij,p} = p \text{ if } y^*_{ij,p} = \begin{cases} \boldsymbol{p}, & y^*_{ij,p} = \max\left(y^*_{ij,1}, y^*_{ij,2}, \ldots, y^*_{ij,P}\right) \\ \mathbf{0}, & \textbf{otherwise} \end{cases} \tag{3.6}$$

where variable definitions are also similar to the continuous case and the ordinal case. The difference is the number of equations: there is only one equation in the continuous and ordinal cases while there are $P-1$ equations in a multinomial case with P possible choices. As a result, each variable is denoted by one additional subscript p in the multinomial case. In a standard multinomial probit model, a base case is first defined and each of the $(P-1)$ equations captures the difference in choice utility between this choice and the base choice. The proposed model implements the same strategy.

The error terms are assumed to follow a multivariate normal distribution.

$$\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{P-1} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \omega_{11} & \cdots & \omega_{1,P-1} \\ \vdots & \ddots & \vdots \\ \omega_{P-1,1} & \cdots & \omega_{P-1,P-1} \end{bmatrix}\right) \tag{3.7}$$

In practice, one of the diagonal elements in the variance-covariance term is often constrained as 1 due to identification concerns. Without such a constraint, the estimated values of the variance-covariance element could be scaled up.

3.3 Connection Between the Two Equations

As only matched pairs could make joint decisions, samples in the matching equation are different from samples in the joint decision making equation. Samples in the matching equation are all possible pairs of supplier i and customer j so that the total number of samples is $I \times J$. However, the samples in the joint decision making equation are just a proportion of $I \times J$ where $ij \in N_1$.

Without considering the effect of the matching process on the joint decision making process, the estimation of θ and β in the joint decision making equation would be biased. The bias can be demonstrated by the conditional expectation of y_{ij}

$$E\left(y_{ij} \mid i, j \text{ are matched}\right) = \beta x + E\left(\varepsilon_{ij} \mid \eta_{ij,c} < \overline{u_{ij}} - \alpha w, \eta_{ij,m} > \underline{u_{ij}} - \alpha w\right) \tag{3.8}$$

where $\eta_{ij,c}$ and $\eta_{ij,m}$ denote the error terms in the matching equation for unmatched pairs and matched pairs respectively. If ε_{ij} is independent to both $\eta_{ij,c}$ and $\eta_{ij,m}$, the estimation of θ and β is unbiased, because the expectation on the right side is zero. However, if ε_{ij} is dependent on any one of $\eta_{ij,c}$ and $\eta_{ij,m}$, the expectation is not zero so that the estimation would be biased. Such a type of estimation bias is usually called a sample selection bias.

Dealing with the sample selection bias can borrow the idea of specifying a sample selection model. Basically, sample selection models assume the error terms of both equations to follow a joint distribution. For the continuous and ordinal outcome cases, the joint distribution is a bivariate normal distribution of

$$\begin{pmatrix} \varepsilon_{ij} \\ \eta_{ij} \end{pmatrix} \sim \Phi(0,\Sigma) = \Phi\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}\right) = \Phi\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}\right) \tag{3.9}$$

Such a specification allows for correlation between the two equations. Using empirical data, the joint distribution can be estimated. If σ_{12} is estimated as zero, the sample selection model reduces to two-part models and the estimation is no longer biased. If σ_{12} is significantly different from zero, the correlation (e.g., sample selection bias) has to be considered in estimation. Note that the variance-covariance matrix is symmetric (e.g., $\sigma_{12} = \sigma_{21}$) and the variance of η is assumed to be one (e.g., $\sigma_{22} = 1$ due to the scaled-up concern). The number of parameters to be estimated in the variance-covariance term is two.

Note that the existing literature specifies the variance-covariance matrix as

$$\begin{bmatrix} 1+\delta^2 & \delta \\ \delta & 1 \end{bmatrix} \tag{3.10}$$

with only one free variable to be estimated. This specification presumes that the variance of the outcome is greater than the variance of the mutual selection process. In contrast, the proposed variance-covariance matrix does not make this assumption, giving the model a higher flexibility.

The multinomial outcome case further complicates the joint distribution of error terms because the joint decision making is composed of multiple equations. This dissertation specifies the joint distribution as a *P*-dimensional multivariate normal distribution

$$\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{P-1} \\ \eta \end{pmatrix} \sim \Phi(0,\Sigma) = \Phi\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1,P-1} & \sigma_{1,P} \\ \vdots & \ddots & \vdots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} & \sigma_{P-1,P} \\ \sigma_{P,1} & \cdots & \sigma_{P,P-1} & 1 \end{bmatrix} \right) \tag{3.11}$$

The elements in the upper-left $(P-1)\times(P-1)$ sub-matrix capture the correlation among the $(P-1)$ choices. The elements in the last row and the last column capture the correlation between a certain choice and the matching equation. The choice-matching correlations could vary across choices, leading to a high flexibility.

3.4 Model Estimation

This dissertation employs a Bayesian MCMC approach with data augmentation to estimate parameters in the proposed model. The reasons of using such a method are (1) The matching relationship is too complex to be disentangled using traditional maximum likelihood estimation approaches: the likelihood of one pair is dependent on the likelihood of all other possible pairs. (2) Latent variables are defined as the dependent variables in the matching equation and discrete outcome joint decision making equations. Data augmentation can deal with latent variables by treating them as parameters.

As discrete cases are extensions of continuous cases, the posterior distributions are discussed in terms of ordinal and multinomial cases.

3.4.1 Introduction to Bayesian MCMC Approach

Based on the conditional probability theory, Bayesian MCMC approach uses prior distributions and likelihood functions to draw random numbers to simulate posterior distributions. With a sufficient number of iterations, the true posterior distributions can be obtained. A general mathematical expression of Bayes rule is

$$f(\vartheta \mid y,u) \propto f(y,u \mid \vartheta)\, f(\vartheta) \tag{3.12}$$

The term ϑ is a collection of parameters. In the linear case, $\vartheta = \{\alpha, \beta, \sigma_{11}, \sigma_{12}\}$. The prior distribution $f(\vartheta)$ is usually defined by researchers. Coupled with the likelihood function $f(y, u \mid \vartheta)$, the posterior distribution can be obtained theoretically. In practice, the posterior distribution does not often belong to any well-known parametric distributions. As a consequence, conjugate priors are usually selected and a number of approximation methods have been proposed to simulate the posterior distribution. One of the most commonly used approaches is Gibbs sampling with data augmentation. The idea is to iteratively draw the values of parameters and latent variables until convergence is reached.

A flowchart of the Bayesian MCMC approach is shown in Figure 3.1 to illustrate the procedure of estimation. The procedure starts with researcher-defined initial parameter values and empirical data. The superscript indicates the number of iterations. Then, the simulation starts. In each iteration, the parameters and latent variables are updated by being randomly sampled based on their probability distributions conditional on the values of all other parameters. The simulation is not terminated until convergence is found. In practice, the researchers usually pre-define a maximum number of iterations. If this number is reached, the iteration is terminated.

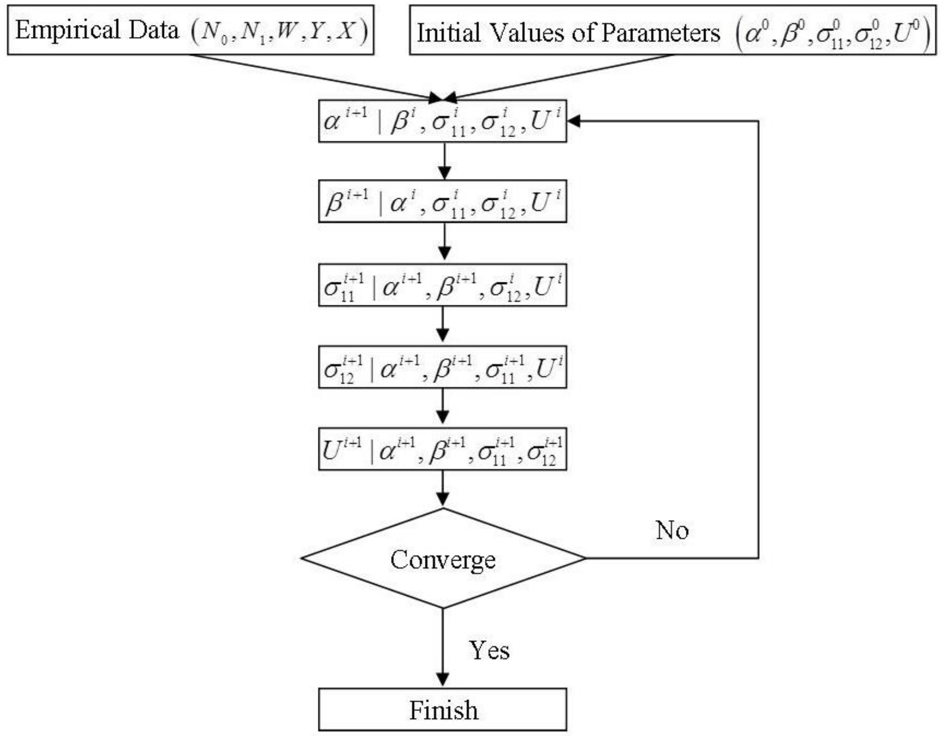


Figure 3.1. Flow chart of Bayesian MCMC with data augmentation

3.4.2 Likelihood Function and Prior Distribution

The likelihood function of the proposed model in the continuous case is given by

$$f(y,u \mid \vartheta) = \prod_{ij \in N_0} \mathrm{I}\left(u_{ij} < \overline{u_{ij}}\right)\phi(\alpha w, 1)$$

$$\times \prod_{ij \in N_1} \mathrm{I}\left(u_{ij} > \underline{u_{ij}}\right)\phi\left(\alpha w + \frac{\sigma_{12}}{\sigma_{11}}\left(y_{ij} - \beta x\right), 1 - \frac{\sigma_{12}^2}{\sigma_{11}}\right) \tag{3.13}$$

$$\times \phi\left(\beta x + \sigma_{12}\left(u_{ij} - \alpha w\right), \sigma_{11} - \sigma_{12}^2\right)$$

The following conjugate prior distributions are used for each parameter: $\alpha \sim Normal\left(\alpha_0, \mathrm{A}_0\right)$, $\beta \sim Normal\left(\beta_0, \mathrm{B}_0\right)$, and $\Sigma \sim Inverse\,Wishart\left(\rho R, \rho\right) I\left(\sigma_{PP} = 1\right)$. The prior of Σ is an inverse Wishart distribution with one on the last diagonal element.

In the ordinal cases, there is one additional set of parameters, threshold μ. Due to the consideration of scale-up issues, one of the thresholds is set constant. Prior distributions of other thresholds are assumed to be constant. To ease the complexity raised by the limited dependent variable, an additional set of parameters y_{ij}^{*} is added and simulated in each iteration. The treatment is similar to the matching utility. The likelihood function is

$$f\left(y=C,u\mid\vartheta\right)=\prod_{ij\in N_0}\mathrm{I}\left(u_{ij}<\overline{u_{ij}}\right)\phi\left(\alpha w,1\right)$$

$$\times\prod_{ij\in N_1}\mathrm{I}\left(u_{ij}>\underline{u_{ij}}\right)\phi\left(\alpha w+\frac{\sigma_{12}}{\sigma_{11}}\left(y_{ij}^{*}-\beta x\right),1-\frac{\sigma_{12}^{2}}{\sigma_{11}}\right) \tag{3.14}$$

$$\times\left(\Phi\left(\mu_C-\beta x+\sigma_{12}\left(u_{ij}-\alpha w\right),\sigma_{11}-\sigma_{12}^{2}\right)-\Phi\left(\mu_{C-1}-\beta x+\sigma_{12}\left(u_{ij}-\alpha w\right),\sigma_{11}-\sigma_{12}^{2}\right)\right)$$

The multinomial cases do not add additional parameters. The prior distributions are also the same as those in the continuous and ordinal cases. The likelihood function is

$$f\left(y=p,u\mid\vartheta\right)=\prod_{ij\in N_0}\mathrm{I}\left(u_{ij}<\overline{u_{ij}}\right)\phi\left(\alpha w,1\right)$$

$$\times\prod_{ij\in N_1}\mathrm{I}\left(u_{ij}>\underline{u_{ij}}\right)\phi\left(\alpha w+\frac{\sigma_{12}}{\sigma_{11}}\left(y_{ij}^{*}-\beta x\right),1-\frac{\sigma_{12}^{2}}{\sigma_{11}}\right) \tag{3.15}$$

$$\times\phi\left(\beta x+\sigma_{12}\left(u_{ij}-\alpha w\right),\sigma_{11}-\sigma_{12}^{2}\right)$$

$$\times\left(I\left(y_{ij}=0\right)I\left(\max\left(y_{ij,p}^{*}\leq 0\right)\right)+\sum_{q=1}^{P}I\left(y_{ij}=P\right)I\left(y_{ij}^{*}>\max\left(0,y_{ij,-q}^{*}\right)\right)\right)$$

3.4.3 Posterior Distribution

Given prior distributions and likelihood functions, posterior distributions of parameters and latent variables can be derived. As discrete cases are extensions of continuous cases, posterior distributions are discussed for ordinal and multinomial cases only. The Gibbs sampling procedure is presented as follows and conducted in MATLAB.

3.4.3.1 Ordinal Cases

There are six steps to simulate posterior distributions for the ordinal cases. These six steps are processed iteratively until convergence is reached.

1. Sample y_{ij}^*. This latent variable follows the same distribution as ε_{ij}. According to multivariate statistics theory, the mean and variance can be derived. Finally, y_{ij}^* follows a normal distribution of

$$y_{ij}^* \sim N\left(\beta x + \sigma_{12}\left(u_{ij} - \alpha w\right), \sigma_{11} - \sigma_{12}^2\right) \tag{3.16}$$

truncated between $\left(\mu_{y_{ij}-1}, \mu_{y_{ij}}\right)$. This dissertation employs the MATLAB code of a truncated multivariate normal distribution written by Robert (1995).

2. Sample u_{ij}. The latent variable outcome in the matching equation also follows a normal distribution. For unmatched pairs, as agents do not make joint decisions, the joint decision making equation does not affect this latent variable in the matching equation. Therefore, for unmatched pairs, the posterior distribution is simply

$$u_{ij} \sim N\left(\alpha w, 1\right) \tag{3.17}$$

truncated above at $\overline{u_{ij}}$.

For matched pairs, the matching equation is affected by the joint decision making equation. The resulted posterior distribution is

$$u_{ij} \sim N\left(\alpha w + \frac{\sigma_{12}}{\sigma_{11}}\left(y_{ij}^* - \beta x\right), 1 - \frac{\sigma_{12}^2}{\sigma_{11}}\right) \tag{3.18}$$

truncated below at $\underline{u_{ij}}$.

3. Sample β. The posterior distribution of parameters in the joint decision making is a normal distribution of

$$\beta \sim N\left(D_\beta d_\beta, D_\beta\right) \tag{3.19}$$

where

$$D_\beta = \left(B_0^{-1} + \frac{1}{\sigma_{11} - \sigma_{12}^2} \sum_{ij \in N_1} x'x \right)^{-1},$$

$$d_\beta = B_0^{-1}\beta_0 + \frac{1}{\sigma_{11} - \sigma_{12}^2} \sum_{ij \in N_1} x\left(y_{ij}^* - \sigma_{12}\left(u_{ij} - \alpha' w \right) \right).$$

4. Sample α. The posterior distribution of parameters in the matching equation is a normal distribution of

$$\alpha \sim N\left(D_\alpha d_\alpha, D_\alpha \right) \tag{3.20}$$

where

$$D_\alpha = \left(A_0^{-1} + \sum_{ij \in N_0} w'w + \left(1 - \frac{\sigma_{12}^2}{\sigma_{11}} \right)^{-1} \sum_{ij \in N_1} w'w \right)^{-1},$$

$$d_\alpha = A_0^{-1}\alpha_0 + \sum_{ij \in N_0} xu_{ij} + \left(1 - \frac{\sigma_{12}^2}{\sigma_{11}} \right)^{-1} \sum_{ij \in N_1} w\left(u_{ij} - \frac{\sigma_{12}}{\sigma_{11}}\left(y_{ij}^* - \beta x \right) \right).$$

5. Sample Σ. The posterior distribution of the variance-covariance matrix is a conditional inverse Wishart distribution

$$\Sigma \sim IW\left(\rho R + \sum_{ij \in N_1} e_{ij} e_{ij}^{'}, N + \rho \right) I\left(\sigma_{22}^2 = 1 \right) \tag{3.21}$$

where $e_{ij} = \begin{bmatrix} \varepsilon_{ij} \\ \eta_{ij} \end{bmatrix}$ and N is the number of matched pairs. Note that such a random matrix is conditional on the value of last diagonal element. In addition, the matrix has to be positive definite. The algorithm of drawing random numbers from such a type of inverse Wishart distributions comes from Nobile (2000).

6. Sample μ_C. The thresholds in the joint decision making equation follow uniform distributions of

$$\mu_C \sim U\left(\max_{y_{ij}=C-1} y_{ij}^*, \min_{y_{ij}=C} y_{ij}^*\right) \tag{3.22}$$

Note that one of the thresholds is set as a constant value due to the scaled-up issue. In this dissertation, $\mu_1 = c$.

3.4.3.2 Multinomial Cases

The estimation process of multinomial cases is similar to the process of ordinal cases. The only difference is the number of simultaneous equations (e.g., two and P, respectively). There are five steps to simulate posterior distributions for the multinomial cases.

1. Sample y_{ij}^*. The latent variable in the joint decision making equation follows a $(P-1)$-dimensional multivariate normal distribution. The posterior distribution is a truncated multivariate normal distribution

$$y_{ij}^* \sim MTN_{R_{ij}(y_{ij})}\left(\delta_{y_{ij}^*}, \Omega_{y_{ij}^*}\right) \tag{3.23}$$

where

$$\delta_{y_{ij}^*} = \begin{bmatrix} \beta_1 x_1 \\ \vdots \\ \beta_{P-1} x_{P-1} \end{bmatrix} + \begin{bmatrix} \sigma_{1P} \\ \vdots \\ \sigma_{P-1,P} \end{bmatrix} \times \left(u_{ij} - \alpha w\right),$$

$$\Omega_{y_{ij}^*} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1,P-1} \\ \vdots & \ddots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} \end{bmatrix} - \begin{bmatrix} \sigma_{1P} \\ \vdots \\ \sigma_{P-1,P} \end{bmatrix} \times \begin{bmatrix} \sigma_{P1} & \cdots & \sigma_{P,P-1} \end{bmatrix}.$$

$R_{ij}(y_{ij})$ denotes the truncated region. If $y_{ij} = 0$, R_{ij} consists of the region where each component of y_{ij}^* is negative. When $y_{ij} = p$, R_{ij} restricts $y_{ij,p}^*$ to be positive and greater than all other $y_{ij,p}^*$.

2. Sample u_{ij}. This latent variable in the matching equation follows a normal distribution. For unmatched pairs, the posterior distribution is exactly the same as that in the ordinal case. For matched pairs, the posterior distribution is

$$u_{ij} \sim N\left(\delta_{u_{ij}}, \Omega_{u_{ij}}\right) \tag{3.24}$$

where

$$\delta_{u_{ij}} = \alpha w + \left[\sigma_{P1}, \cdots, \sigma_{P,P-1}\right] \times \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1,P-1} \\ \vdots & \ddots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} \end{bmatrix}^{-1} \times \begin{bmatrix} y^{*}_{ij,1} - \beta_1 x_1 \\ \vdots \\ y^{*}_{ij,P-1} - \beta_{P-1} x_{P-1} \end{bmatrix},$$

$$\Omega_{u_{ij}} = 1 - \left[\sigma_{P,1}, \cdots, \sigma_{P,P-1}\right] \times \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{P,P-1} \\ \vdots & \ddots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} \end{bmatrix}^{-1} \times \begin{bmatrix} \sigma_{1,P} \\ \vdots \\ \sigma_{P-1,P} \end{bmatrix}$$

The distribution is truncated below at $\underline{u_{ij}}$.

3. Sample β_p. Parameters in each of joint decision making equation follow a normal distribution of

$$\beta_p \sim N\left(D_{\beta_p} d_{\beta_p}, D_{\beta_p}\right) \tag{3.25}$$

where

$$D_{\beta_p} = \left(B_0^{-1} + \sum_{ij \in N_1} x_p^{'} \Sigma_p^{-1} x_p\right)^{-1},$$

$$d_{\beta_p} = B_0^{-1} \beta_{p,0} + \sum_{ij \in N_1} x_p \Sigma_p^{-1} \left(y^{*}_{ij,p} - \sigma_{1,(-p)} \times \sigma^{-1}_{(-p),(-p)} \times \begin{bmatrix} y^{*}_{ij,1} - \beta_1 x_1 \\ \vdots \\ y^{*}_{ij,P-1} - \beta_{P-1} x_{P-1} \\ u_{ij} - \alpha w \end{bmatrix} \right)_{(-p)}$$

with $\Sigma_p = \sigma_{pp} - \sigma_{1,(-p)} \times \sigma^{-1}_{(-p),(-p)} \times \sigma_{(-p),1}$.

4. Sample α. The posterior distribution of parameters in the matching equation is a normal distribution of

$$\alpha \sim N\left(D_\alpha d_\alpha, D_\alpha\right) \tag{3.26}$$

where

$$D_\alpha = \left(A_0^{-1} + \sum_{ij\in N_0} ww' + \sum_{ij\in N_1} w\Sigma_u^{-1} w'\right),$$

$$d_\alpha = \left(A_0^{-1}\alpha_0 + \sum_{ij\in N_0} wu_{ij} + \sum_{ij\in N_1} w\Sigma_u^{-1}\left(u_{ij} - \sigma_{1,(-P)} \times \sigma_{(-P),(-P)}^{-1} \times \begin{bmatrix} y_{ij,1}^* - \beta_1 x_1 \\ \vdots \\ y_{ij,P-1}^* - \beta_{P-1} x_{P-1} \end{bmatrix}\right)\right)$$

with $\Sigma_P = 1 - \sigma_{1,(-P)} \times \sigma_{(-P),(-P)}^{-1} \times \sigma_{(-P),1}$.

5. Sample Σ. The posterior distribution of the variance-covariance matrix is a conditional inverse Wishart distribution

$$\Sigma \sim IW\left(\rho R + \sum_{ij\in N_1} e_{ij} e_{ij}^{'}, N + \rho\right) I\left(\sigma_{PP}^2 = 1\right) \tag{3.27}$$

where $e_{ij} = \begin{bmatrix} \varepsilon_{ij,1} \\ \vdots \\ \varepsilon_{ij,P-1} \\ \eta_{ij} \end{bmatrix}$ and N is the number of matched pairs.

4. Model Validation and Sensitivity Analysis

Given the specification of the proposed model, this chapter first presents a model validation analysis to show the performance of the proposed model. The validation is implemented using simulation data that are randomly generated based on pre-defined parameters. With the simulation data, the proposed estimation approach is implemented and the parameter recovery capability is analyzed as an indicator of model performance. Then, a sensitivity analysis is followed to show model performance at a variety of parameter values. For each parameter value, the simulation data are randomly generated for 30 times and the parameter recovery capability is analyzed correspondingly to illustrate the sensitivity of the estimation approach. The validation and sensitivity analysis are discussed for the ordinal and multinomial cases, respectively.

4.1 Model Validation

The model validation analysis is implemented as the following steps: (1) Define parameter values; (2) Use defined parameter values to generate simulation data; (3) Treat the defined parameters as unknown and use the simulation data to estimate these parameters; (4) Compare the estimated parameters and the pre-defined parameters. If a good parameter recovery capability is found, the model is successfully validated.

First, the parameters in the matching equation are defined and identical for ordinal and multinomial cases. The assumed market is two-side and each side has 50 agents (i.e., $I = 50$ and $J = 50$). Therefore, there are 2,500 (i.e., $I \times J = 2,500$) possible pairs, leading to 2,500 pairwise utility. Each agent could have at most 25 collaborators on the other side of the market. (The number of collaborators is driven by the randomly generated data. Some agents may turn out to have less than 25 collaborators.) Which two agents are matched together depends on the relative magnitude of pairwise utility, which is determined by three explanatory variables and error terms. The explanatory variables are attributes of each side, respectively (e.g., w_i and w_j which are constant over one certain agent). The explanatory variables also include a joint factor (e.g., w_{ij}, which varies across pairs). These explanatory variables are generated by standard normal distributions. Table 4.1 summarizes values of parameters in the matching equation.

Table 4.1. Parameter values in the matching equation

Parameter	True Values	Initial Values
I	50 (at most 25 matched pairs)	
J	50 (at most 25 matched pairs)	
α	(-0.6, 0.9, -0.3)	(0, 0, 0)

4.1.1 Ordinal Case

The simulation data in the joint decision making equation contains three ordinal categories, leading to two threshold variables. Three explanatory variables are assumed to determine the latent variable in the joint decision making outcome. Similarly, two of them are attributes of each side, respectively (e.g., x_i and x_j which are constant over one certain agent). The other explanatory variable (i.e., x_{ij}) is the joint factor, which varies across pairs. These explanatory variables are also generated by standard normal distributions. The error terms in the matching equation and joint decision making equation are assumed to follow a bi-variate normal distribution of

$$\begin{pmatrix} \varepsilon_{ij} \\ \eta_{ij} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1.5 & -0.3 \\ -0.3 & 1 \end{bmatrix} \right) \tag{4.1}$$

Table 4.2 summarizes values of parameters in the ordinal case of the joint decision making equation and the variance-covariance matrix.

Table 4.2. Parameter values in the joint decision making equation and variance-covariance matrix for the ordinal case

Parameter	True Values	Initial Values
β	(0.3, 0.6, -0.9)	(0, 0, 0)
μ	(-0.5, 0.5)	(fix at -0.5, 0)
σ_{11}	1.5	1
σ_{12}	-0.3	0

Based on these pre-defined values of parameters, the simulation data are generated randomly. This simulation data turns out to have 1,220 matched pairs.

Table 4.3 summarizes the size, mean, standard deviation, minimum value, and maximum values of the generated data.

Table 4.3. Parameter values of the ordinal case for performance test

Generated Data	Size	Mean	Standard Deviation	Min	Max
w_i	$I \times J = 2,500$	-0.05	1.00	-2.31	1.83
w_j	$I \times J = 2,500$	-0.01	0.89	-1.85	1.96
w_{ij}	$I \times J = 2,500$	-0.03	1.00	-3.75	3.09
x_i	1,220	-0.08	0.99	-2.31	1.83
x_j	1,220	0.03	0.87	-1.85	1.96
x_{ij}	1,220	-0.23	0.99	-3.75	2.74
y_{ij}	1,220		$y_{ij} = 1$, 464 $y_{ij} = 2$, 293 $y_{ij} = 3$, 463		

Then, the proposed Bayesian MCMC approach is used to estimate the parameters with the input of the generated data. The estimation process starts with the initial values specified in Table 4.1 and Table 4.2. If the estimated results are close to the pre-defined values, the model is validated. Finally, the estimation procedure is implemented by iterating 6,000 times with the first 4,000 as the burn-in period, which allows for parameters to gradually move from the initial values to true values. The traces of estimated parameters are shown in Figure 4.1 with the x-axle as iteration numbers and the y-axle as parameter values. In addition, the black straight line indicates true values of corresponding parameters.

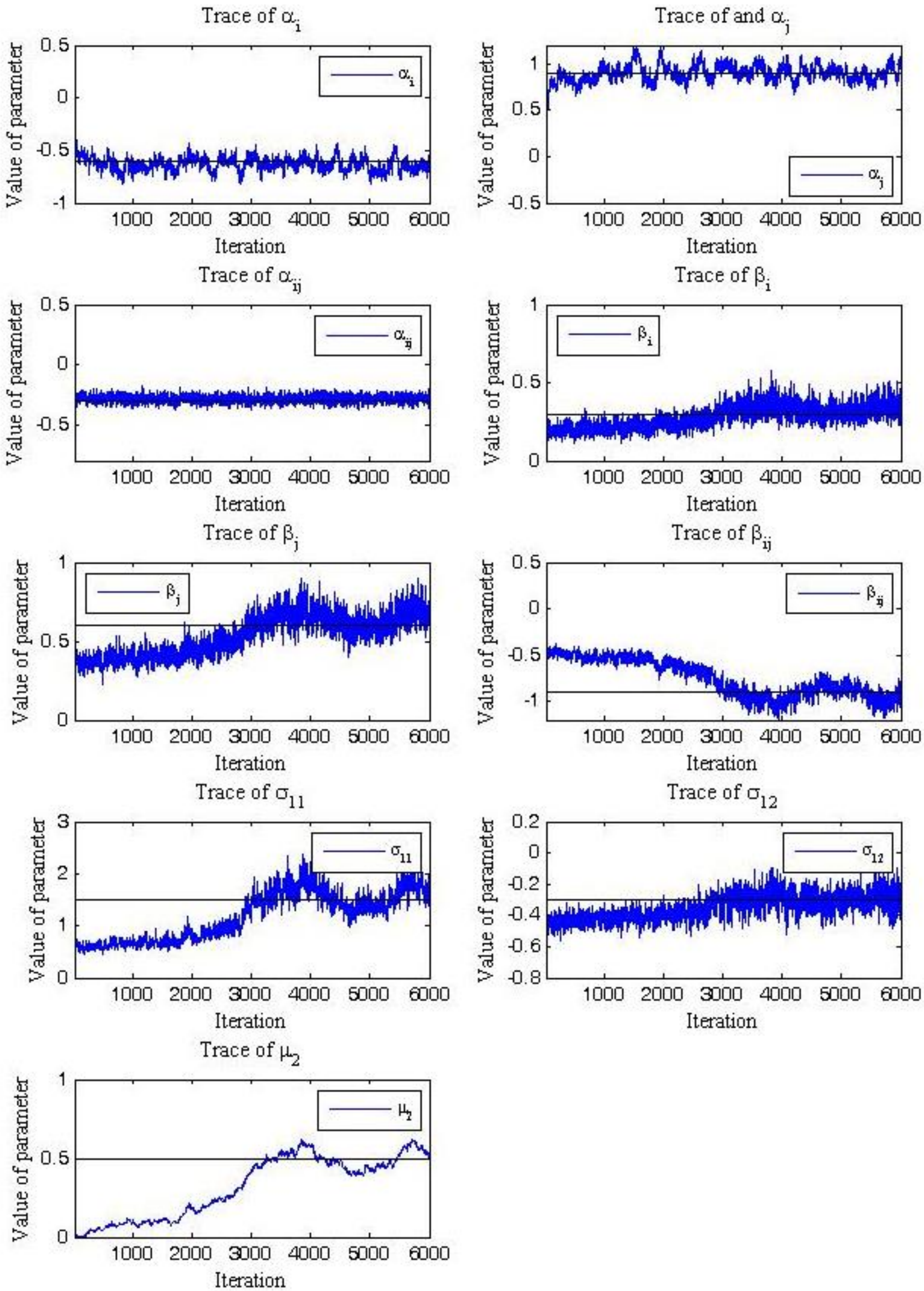


Figure 4.1. Estimated parameter traces of the ordinal case for model validation

Figure 4.1 shows that the estimated values gradually move towards the true values from the initial values for all parameters. The estimated values start shifting around the true values at about 4,000 iterations. Such traces indicate the estimation approach is able to recover the parameter values successfully and thus, the proposed model is validated.

Posterior distributions of estimated parameters are presented in Figure 4.2, which are generated based on the iteration 4,001 to 6,000 of the corresponding parameters. The x-axle denotes values of corresponding parameters and y-axle is the frequency in the investigated 2,000 iterations.

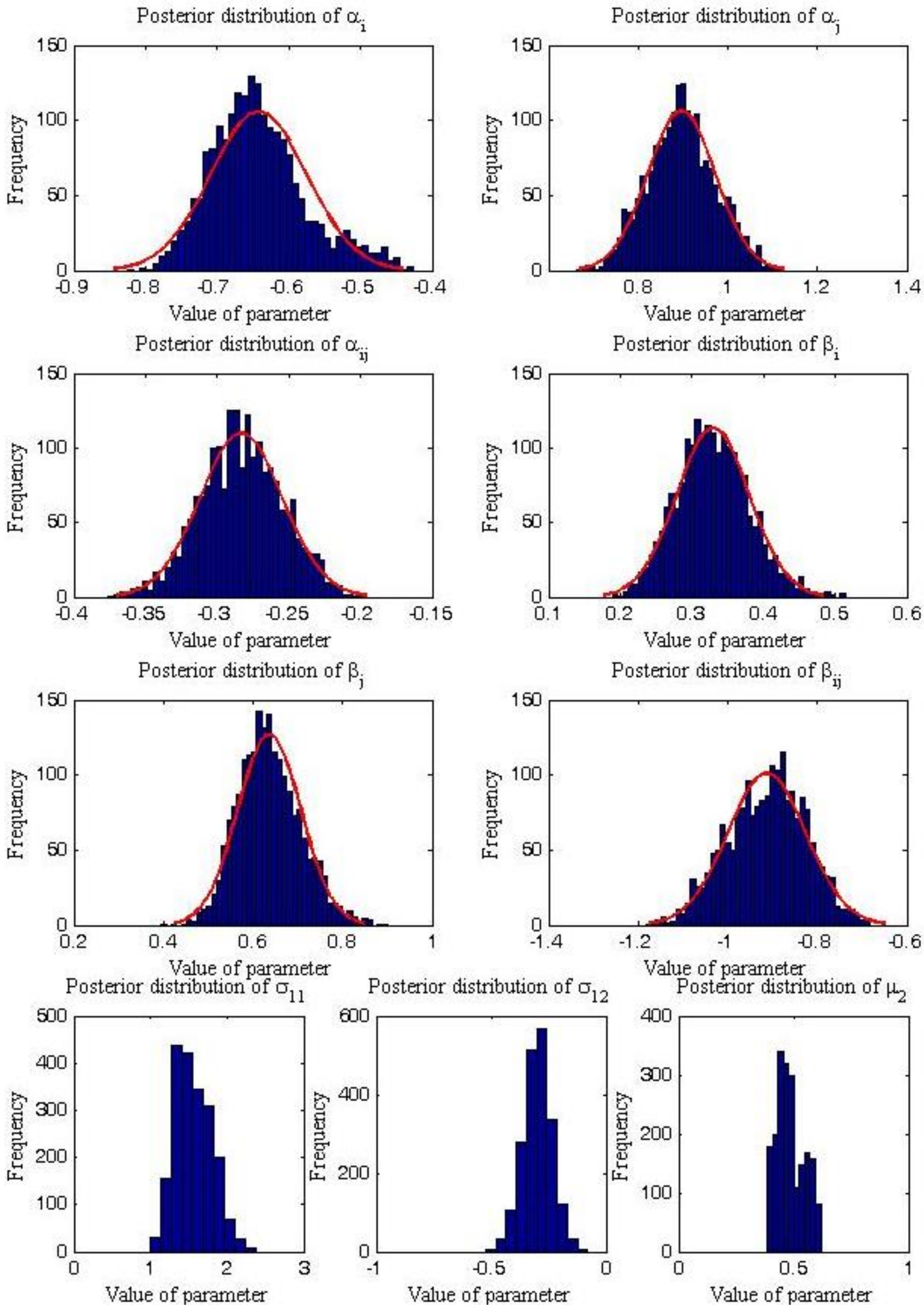


Figure 4.2. Posterior distributions of estimated parameters in the ordinal case

The first six histograms present bell curves, which are consistent with the assumption that their posteriors are normal distributions. The last three histograms also show averages of the true parameter values, recovering the parameter values using simulation data.

The estimation results are further summarized in Table 4.4.

Table 4.4. Estimation summary of simulation data in the ordinal outcome case

Parameter	True Value	Estimated Mean	Standard Deviation	95% CI Low	95% CI Up
α_i	-0.6	-0.64	0.07	-0.77	-0.51
α_j	0.9	0.90	0.08	0.75	1.04
α_{ij}	-0.3	-0.28	0.03	-0.34	-0.23
β_i	0.3	0.33	0.05	0.23	0.43
β_j	0.6	0.64	0.07	0.50	0.77
β_{ij}	-0.9	-0.91	0.09	-1.08	-0.74
σ_{11}	1.5	1.55	0.24	-	-
σ_{12}	-0.3	-0.30	0.06	-	-
μ_2	0.5	0.49	0.06	-	-

Estimation results show that all of the estimated values are close to the true values and the deviations from true values are about the same across all parameters.

Based on the estimation traces, posterior distributions, and error statistics, the proposed estimation method is able to recover parameters. Therefore, the proposed joint response model with ordinal outcome is successfully validated.

4.1.2 Multinomial Case

The multinomial outcome case is assumed to contain three categorical outcomes in the joint decision making equation, leading to two equations. Each equation is assumed to have three explanatory variables. Similarly, two of them (i.e., $x_{i,p}$ and $x_{j,p}$) are attributes of each side, respectively (e.g., the value is constant over one certain agent) and the other one (i.e., $x_{ij,p}$) is the joint factor, which varies across pairs. Note that there can be alternative-specific variables, but they make no difference mathematically. All of the explanatory variables are generated by standard normal distributions. Table 4.5 summarizes values of parameters in the multinomial case. Given the joint decision making has two equations, the error terms follow a tri-variate normal distribution.

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \eta \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1.5 & 0.3 & 0.2 \\ 0.3 & 1.2 & 0.1 \\ 0.2 & 0.1 & 1 \end{bmatrix} \right) \tag{4.2}$$

Table 4.5. Parameter values for the multinomial case in the joint decision making equation

Parameter	True Values	Initial Values
β_1	(-0.9, 0.6, 0.3)	(0, 0, 0)
β_2	(0.6, 0.3, -0.9)	(0, 0, 0)
σ_{11}	1.5	1
σ_{22}	1.2	1
σ_{12}	0.3	0
σ_{23}	0.2	0
σ_{13}	0.1	0

Based on the pre-defined values of parameters, the simulation data of the multivariate case are generated.

Table 4.6 summarizes the average, standard deviation, minimum value, and maximum values of the generated data. Given the sizes of agents on both sides, the number of observations in the matching equation is $I \times J = 2,500$. The generated data leads to 1,221 matched pairs.

Table 4.6. Summary statistics of generated data in the multinomial case

Generated Data	Size	Mean	Standard Deviation	Min	Max
w_i	$I \times J = 2{,}500$	-0.64	1.03	-3.09	1.28
w_j	$I \times J = 2{,}500$	-0.67	1.07	-2.96	2.32
w_{ij}	$I \times J = 2{,}500$	-0.01	1.00	-3.58	3.61
$x_{i,1}$	1,221	-0.08	0.99	-2.31	1.83
$x_{j,1}$	1,221	0.03	0.87	-1.85	1.96
$x_{ij,1}$	1,221	-0.18	0.96	-3.58	2.35
$x_{i,2}$	1,221	-0.08	0.99	-2.31	1.83
$x_{j,2}$	1,221	0.03	0.87	-1.85	1.96
$x_{ij,2}$	1,221	-0.18	0.96	-3.58	2.35

Then, the Bayesian MCMC approach is used to estimate the parameters with the input of the generated data. Due to the faster speed of convergence, the estimation of the multinomial case reduces to iterate 500 times with the first 300 as the burn-in period. The traces of estimated parameters are shown in Figure 4.3.

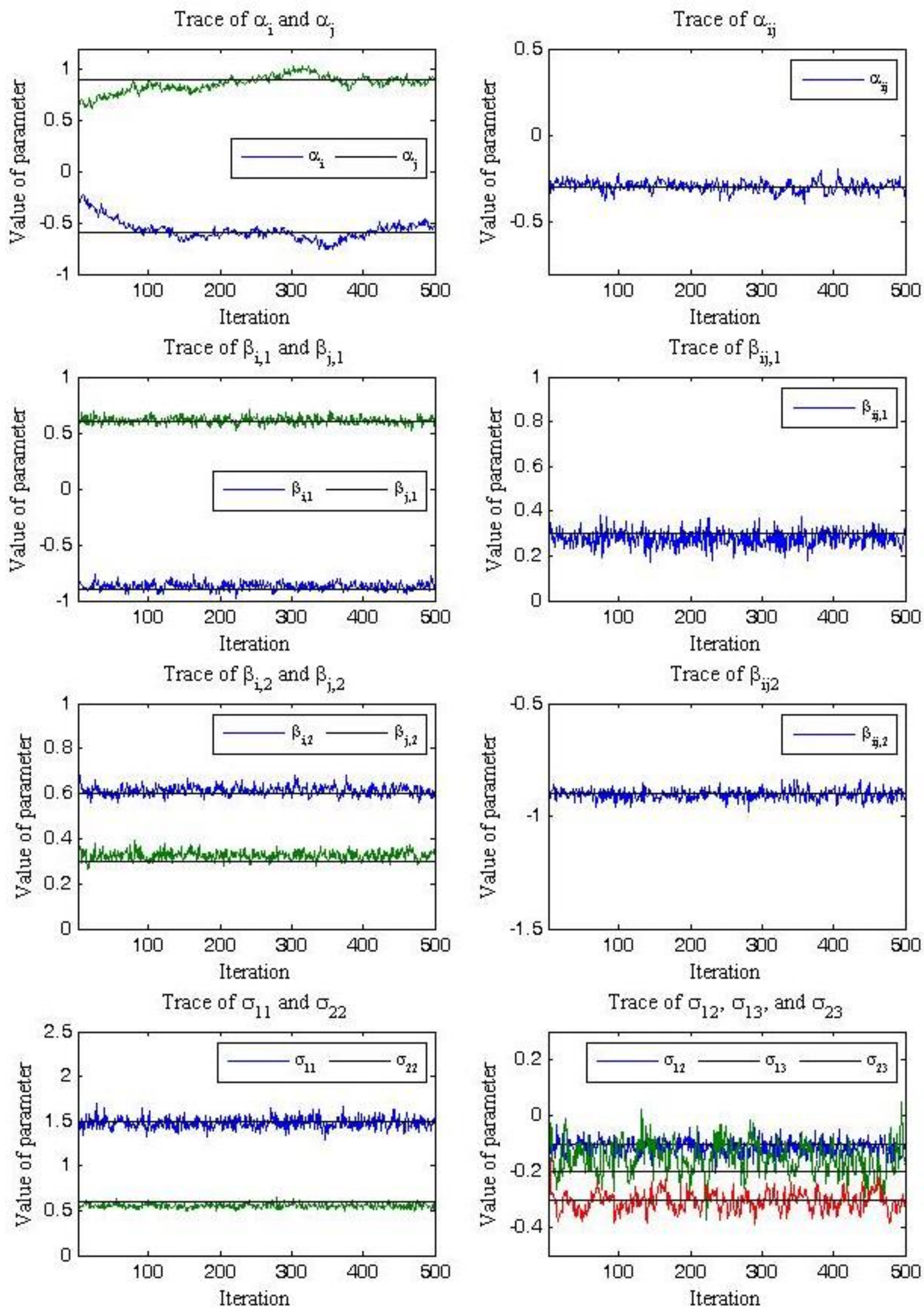
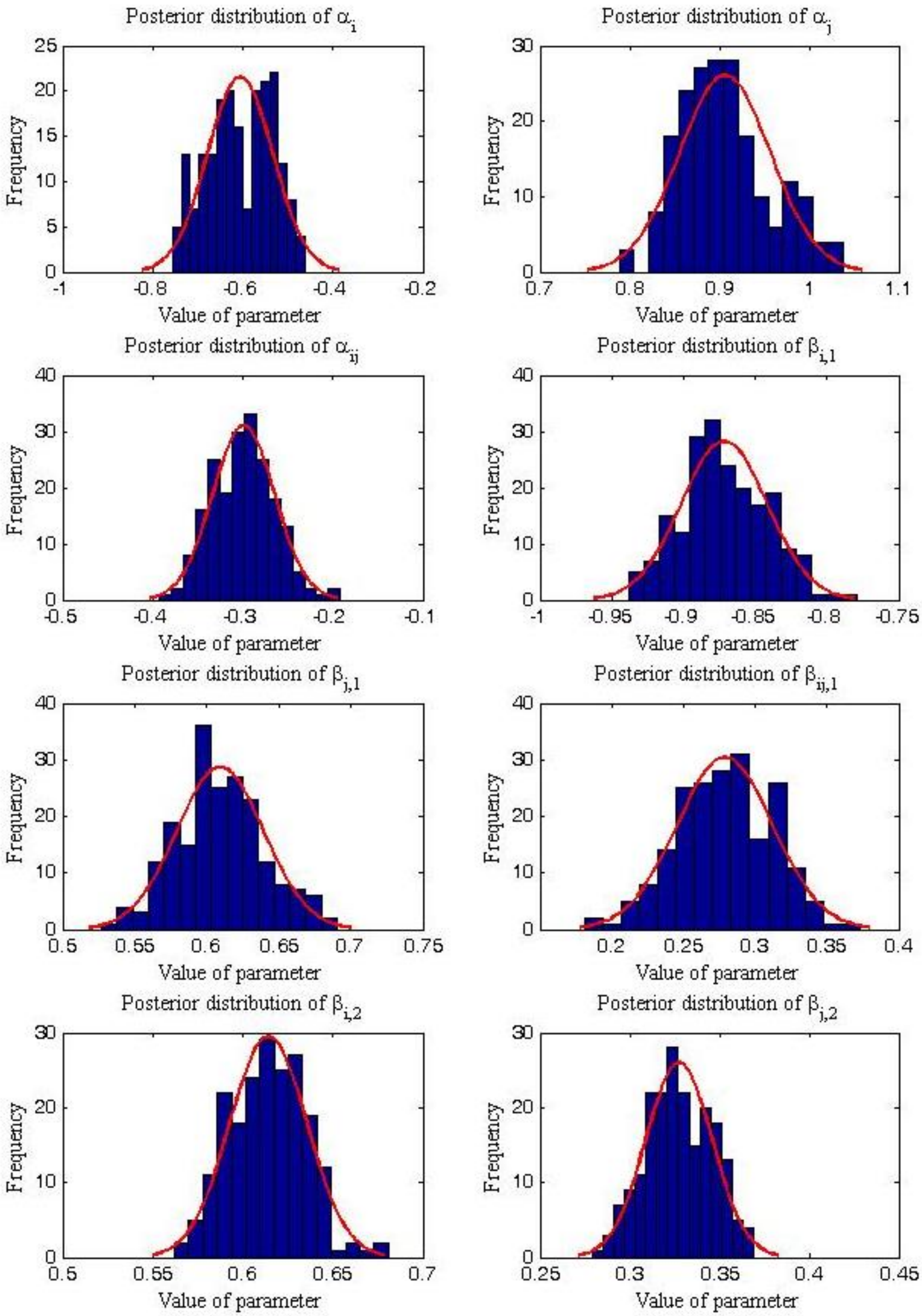


Figure 4.3. Estimated parameter traces of the multinomial case for performance test

Traces show that all parameters converge to their true values fast, which indicates a good parameter recovery capability. Figure 4.4 shows the corresponding posterior distributions.



Posterior distribution of α_i
Posterior distribution of α_j
Posterior distribution of α_{ij}
Posterior distribution of $\beta_{i,1}$
Posterior distribution of $\beta_{j,1}$
Posterior distribution of $\beta_{ij,1}$
Posterior distribution of $\beta_{i,2}$
Posterior distribution of $\beta_{j,2}$
Frequency
Value of parameter

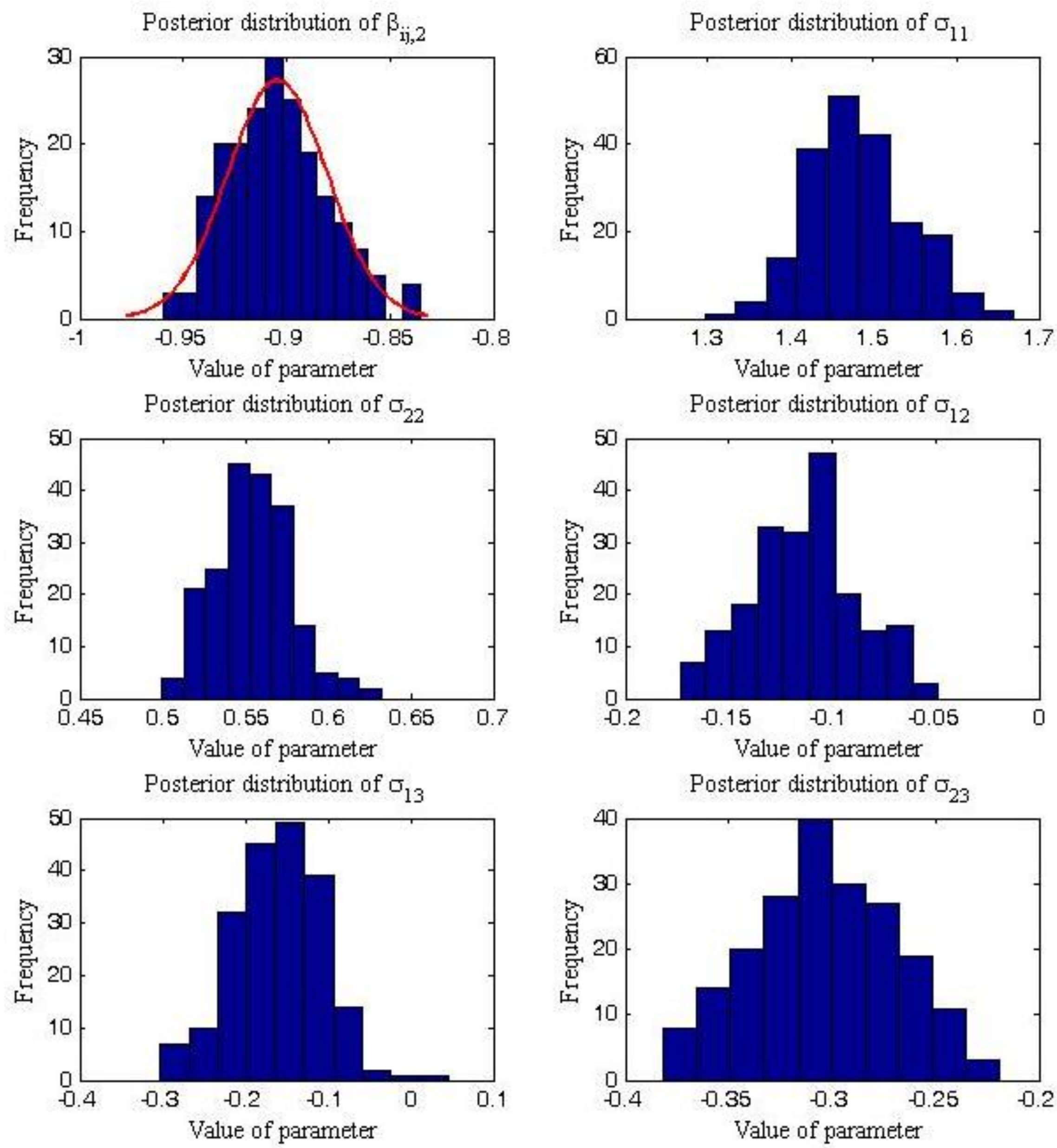


Figure 4.4. Posterior distributions of all estimated parameters in the multinomial case

The estimation results are summarized in Table 4.7.

Table 4.7. Estimation result summary of simulation data in the multinomial outcome case

Parameter	True Value	Mean	Standard Deviation	95% CI Low	95% CI Up
α_i	-0.6	-0.61	0.07	-0.75	-0.46
α_j	0.9	0.91	0.05	0.81	1.01
α_{ij}	-0.3	0.30	0.03	-0.37	-0.23
$\beta_{i,1}$	-0.9	-0.87	0.03	-0.93	-0.81
$\beta_{j,1}$	0.6	0.61	0.03	0.55	0.67
$\beta_{ij,1}$	0.3	0.28	0.03	0.21	0.34
$\beta_{i,2}$	0.6	0.61	0.02	0.57	0.66
$\beta_{j,2}$	0.3	0.33	0.02	0.29	0.36
$\beta_{ij,2}$	-0.9	-0.90	0.02	-0.95	-0.86
σ_{11}	1.5	1.48	0.06	-	-
σ_{22}	0.5	0.56	0.02	-	-
σ_{12}	-0.1	-0.11	0.03	-	-
σ_{13}	-0.2	-0.16	0.05	-	-
σ_{23}	-0.3	-0.30	0.03	-	-

According to the estimation results, all of the estimated values are close to the true values, yielding good parameter recovery. Among them, the performance of parameters for explanatory variables is better than parameters in the variance-covariance matrix.

4.2 Sensitivity Analysis

Validation by only one randomly-generated dataset may not thoroughly evaluate the parameter recovery capability of the proposed estimation approach. Two aspects can be implemented to improve the validation process: measuring the recovery capability at different parameter values and repeating the estimation process by multiple times. Thus, this section uses a variety of different pre-defined parameter values to discuss the sensitivity of estimation.

In order to compare parameter recovery performance, two statistics are introduced, absolute percentage error (APE) and percentage square error (PSE).

$$APE = \sum_{t=1}^{T} \frac{|z_t - z_0|}{|z_0|} / T \tag{4.3}$$

$$PSE = \sum_{t=1}^{T} \left(\frac{z_t - z_0}{z_0} \right)^2 / T \tag{4.4}$$

In the equations, $t\left(t = 1 \ldots T\right)$ denotes the number of iterations. Term z_t is the value of parameter at iteration t and term t_0 is the true parameter value. APE measures the average absolute deviation from the true values. PSE further uses the squared error to penalize large deviations.

4.2.1 Ordinal Case

Table 4.8 lists the scenarios designed for the ordinal case in the sensitivity analysis. Two aspects are given special attention: the matching structure and the variance-covariance matrix. Disentangling the intricate matching structure is one of the main objectives of this dissertation. Therefore, two matching structures are investigated: One has 50 agents on both sides and the other has 10 versus 100 agents on each side, respectively. These two structures can reveal the difference between balanced matching structures and unbalanced matching structures. They can also explore the influence of sample sizes (e.g., 2,500 v.s. 1,000).

The variance-covariance matrix characterizes the sample selection process. In the ordinal case, there is one free variable on the diagonal and one free variable on the off-diagonal. The diagonal element is investigated at 1.5 and 0.6. The off-diagonal element is investigated at 0.3 and -0.3.

Table 4.8. Summary of sensitivity analysis scenarios for the ordinal case

	I	J	Variance-Covariance term
1	50	50	$\begin{pmatrix} 1.5 & 0.3 \\ 0.3 & 1 \end{pmatrix}$
2	50	50	$\begin{pmatrix} 1.5 & -0.3 \\ -0.3 & 1 \end{pmatrix}$
3	50	50	$\begin{pmatrix} 0.8 & 0.3 \\ 0.3 & 1 \end{pmatrix}$
4	50	50	$\begin{pmatrix} 0.8 & -0.3 \\ -0.3 & 1 \end{pmatrix}$
5	10	100	$\begin{pmatrix} 1.5 & 0.3 \\ 0.3 & 1 \end{pmatrix}$
6	10	100	$\begin{pmatrix} 1.5 & -0.3 \\ -0.3 & 1 \end{pmatrix}$
7	10	100	$\begin{pmatrix} 0.8 & 0.3 \\ 0.3 & 1 \end{pmatrix}$
8	10	100	$\begin{pmatrix} 0.8 & -0.3 \\ -0.3 & 1 \end{pmatrix}$

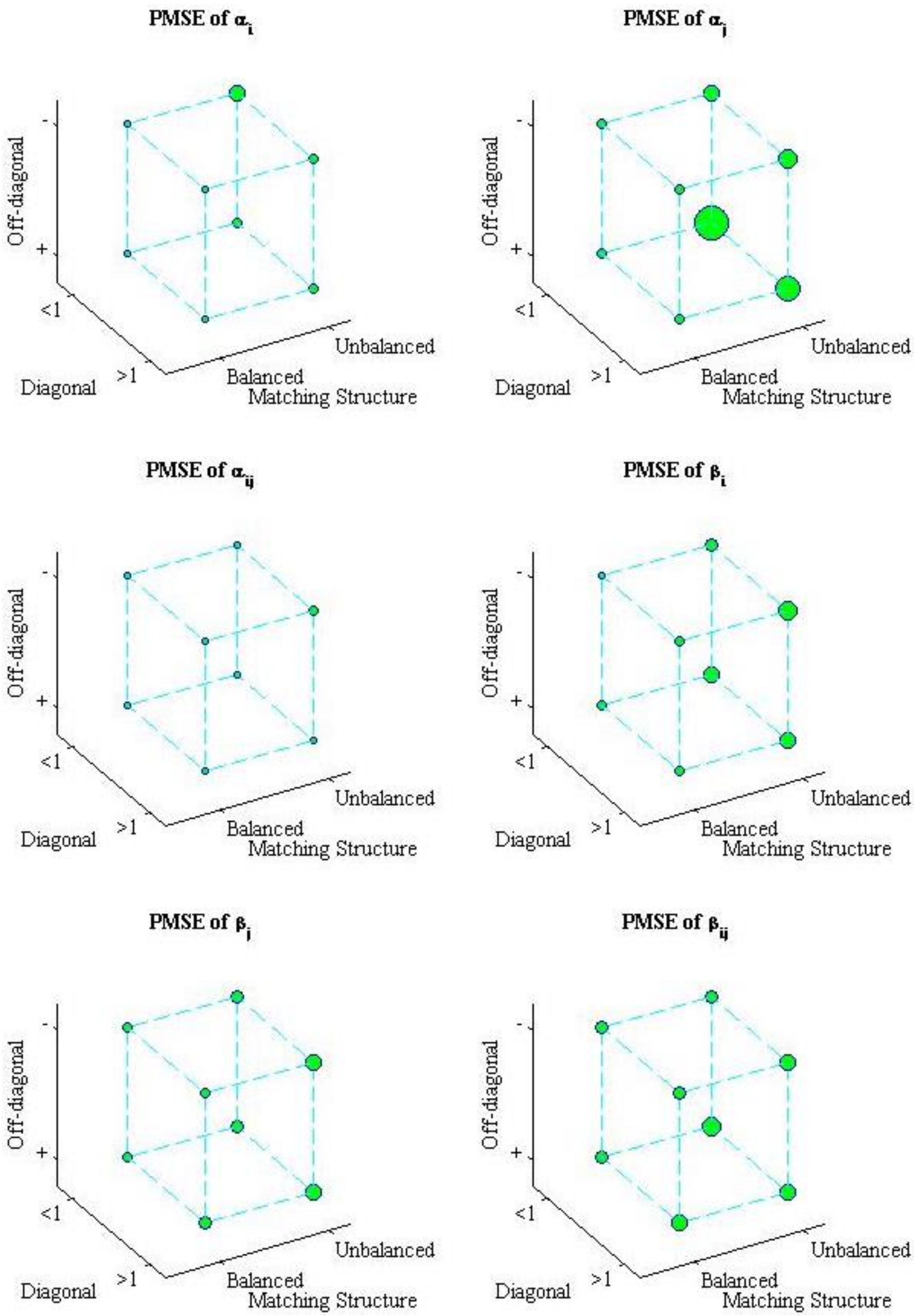
For each scenario, the simulation data is generated 30 times and the estimation is implemented 30 times. The summary of the estimation results and the corresponding error statistics are presented in Table 4.9.

Table 4.9. Sensitivity analysis results for the ordinal case

Scenario	1					2				
Parameter	True Value	Mean	S.D.	MAPE	PMSE	True Value	Mean	S.D.	MAPE	PMSE
α_i	-0.6	-0.62	0.05	6.80	0.49	-0.6	-0.60	0.05	5.98	0.36
α_j	0.9	0.89	0.08	8.51	0.99	0.9	0.90	0.08	7.04	0.85
α_{ij}	-0.3	-0.31	0.03	8.82	0.37	-0.3	-0.31	0.03	8.71	0.38
β_i	0.3	0.30	0.05	18.81	1.65	0.3	0.28	0.05	17.70	1.42
β_j	0.6	0.59	0.06	13.48	1.85	0.6	0.57	0.06	12.43	1.39
β_{ij}	-0.9	-0.88	0.08	14.44	3.09	-0.9	-0.86	0.08	15.22	2.87
μ_2	0.5	0.45	0.05	26.45	5.07	0.5	0.45	0.05	23.33	3.74
σ_{11}	1.5	1.42	0.21	26.18	15.98	1.5	1.46	0.21	24.17	12.86
σ_{22}	0.3	0.25	0.07	44.57	8.22	-0.3	-0.32	0.06	17.75	1.35
Scenario	**3**					**4**				
α_i	-0.6	-0.62	0.06	7.19	0.44	-0.6	-0.61	0.05	6.92	0.50
α_j	0.9	0.90	0.08	7.83	0.88	0.9	0.91	0.08	6.71	0.80
α_{ij}	-0.3	-0.31	0.03	8.15	0.31	-0.3	-0.30	0.03	9.36	0.35
β_i	0.3	0.27	0.04	15.91	1.01	0.3	0.30	0.04	10.87	0.54
β_j	0.6	0.54	0.04	12.33	1.26	0.6	0.62	0.05	11.06	1.17
β_{ij}	-0.9	-0.78	0.06	16.28	2.84	-0.9	-0.92	0.07	11.86	2.07
μ_2	0.5	0.37	0.03	29.24	5.10	0.5	0.51	0.05	20.11	3.03
σ_{11}	0.8	0.64	0.08	27.57	7.40	0.8	0.83	0.11	16.25	3.72
σ_{22}	0.3	0.19	0.05	44.08	7.84	-0.3	-0.29	0.05	13.46	0.84

Scenario			**5**					**6**		
Parameter	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**
α_i	-0.6	-0.57	0.09	11.63	1.11	-0.6	-0.55	0.09	13.71	1.61
α_j	0.9	1.04	0.16	22.75	10.85	0.9	1.06	0.16	22.07	5.86
α_{ij}	-0.3	-0.30	0.05	11.86	0.66	-0.3	-0.30	0.05	13.04	0.78
β_i	0.3	0.34	0.10	30.96	3.78	0.3	0.32	0.10	37.10	5.73
β_j	0.6	0.67	0.12	19.75	3.85	0.6	0.64	0.12	20.94	3.83
β_{ij}	-0.9	-0.95	0.15	15.58	3.14	-0.9	-0.97	0.16	19.08	4.24
μ_2	0.5	0.54	0.11	22.79	3.70	0.5	0.52	0.11	26.37	4.34
σ_{11}	1.5	1.82	0.50	33.05	21.69	1.5	1.77	0.47	36.15	26.95
σ_{22}	0.3	0.40	0.16	60.07	16.35	-0.3	-0.25	0.12	34.52	5.21
Scenario			**7**					**8**		
α_i	-0.6	-0.60	0.10	12.85	1.66	-0.6	-0.59	0.10	17.82	3.10
α_j	0.9	1.06	0.17	26.24	18.09	0.9	0.96	0.16	15.62	3.16
α_{ij}	-0.3	-0.30	0.05	10.71	0.50	-0.3	-0.30	0.05	9.35	0.41
β_i	0.3	0.35	0.08	30.61	4.67	0.3	0.33	0.08	23.37	2.99
β_j	0.6	0.67	0.10	18.23	2.79	0.6	0.67	0.09	18.12	2.99
β_{ij}	-0.9	-1.02	0.13	22.02	5.38	-0.9	-1.01	0.12	15.42	2.90
μ_2	0.5	0.58	0.09	30.66	5.54	0.5	0.60	0.08	28.08	4.75
σ_{11}	0.8	1.07	0.25	46.92	23.06	0.8	1.01	0.21	31.09	11.07
σ_{22}	0.3	0.38	0.12	53.13	12.12	-0.3	-0.25	0.09	23.73	2.69

In Figure 4.5, 3-D scatter plots are used to visually demonstrate the error statistics across all scenarios. Three dimensions represent the matching structures (balanced vs. unbalanced), diagonal elements (greater than 1 vs. smaller than 1), and off-diagonal elements (positive covariance term vs. negative covariance term). The scatter plot uses scale to indicate the magnitude of PMSE.


PMSE of α_i
Off-diagonal
-
+
<1
>1
Diagonal
Balanced
Unbalanced
Matching Structure
PMSE of α_j
Off-diagonal
-
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Diagonal
Balanced
Unbalanced
Matching Structure
PMSE of α_{ij}
Off-diagonal
-
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Matching Structure
PMSE of β_i
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Unbalanced
Matching Structure

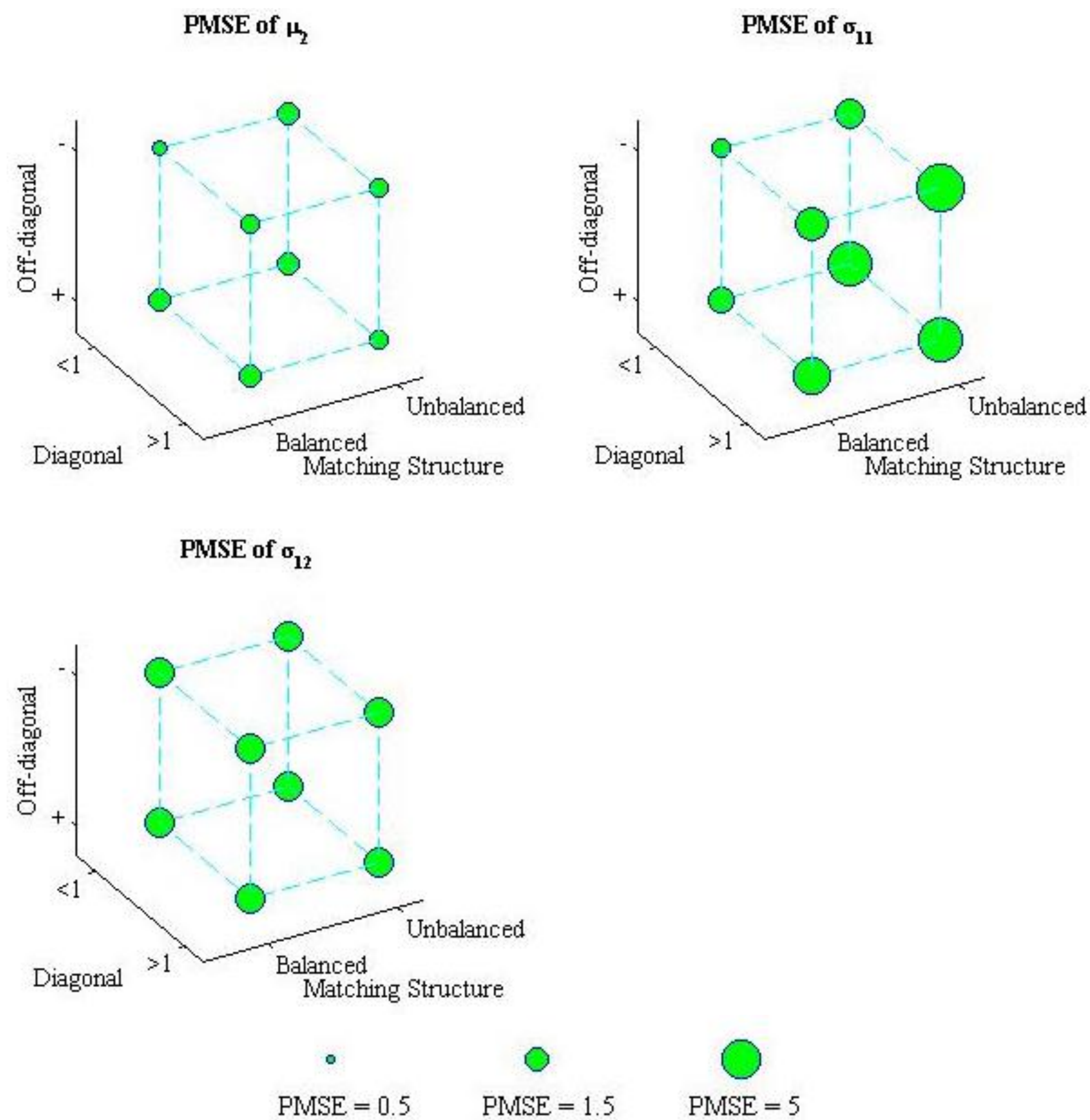


Figure 4.5. PMSE of 30 random samples in the ordinal case

The following findings can be concluded from Figure 4.5. Comparing all scenarios, balanced matching structures turn out to have lower PMSE, indicating a better recovery capability. The reasons can be twofolds: (1) the sample size of balanced matching structure is larger than the sample size of unbalanced matching. A large sample size generally enables good model identification; (2) The simulation data of the unbalanced sample endures less variation (e.g., as $I = 10$, some of the independent variables have only 10 different values), which reduces the identification ability.

The thresholds and variance-covariance matrix elements have larger PMSE than other parameters. Such estimation performance can be attributed to the scale-up issues: thresholds and variance elements could increase or decrease together to improve the fit. Comparing to other linear coefficients, these parameters are expected to have large variances in the estimation.

4.2.2 Multinomial Case

Similarly, two matching structures are investigated in the multinomial case, a balanced matching structure and an unbalanced matching structure. The variance-covariance matrix in the multinomial cases has two free variables on the diagonal elements and three free variables on the off-diagonal elements. The diagonal elements are investigated at (1.5, 1.2), (1.5, 0.6), and (0.8, 0.6). The off-diagonal elements are investigated at (0.3, 0.2, 0.1) and (-0.1, -0.2, -0.3). Table 4.10 summarizes the investigated scenarios.

Table 4.10. Summary of validation scenarios in the multinomial case.

	I	J	Variance-Covariance term
1	50	50	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 1.2 & 0.1 \\ & & 1 \end{pmatrix}$
2	50	50	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 1.2 & -0.3 \\ & & 1 \end{pmatrix}$
3	50	50	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
4	50	50	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$
5	50	50	$\begin{pmatrix} 0.8 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
6	50	50	$\begin{pmatrix} 0.8 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$
7	10	100	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 1.2 & 0.1 \\ & & 1 \end{pmatrix}$
8	10	100	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 1.2 & -0.3 \\ & & 1 \end{pmatrix}$
9	10	100	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
10	10	100	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$
11	10	100	$\begin{pmatrix} 0.8 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
12	10	100	$\begin{pmatrix} 0.8 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$

For each scenario, 30 simulations are randomly generated and estimated using the proposed estimation method. In each estimation process, the first 300 iterations are treated as the "burn-in" period and the last 200 iterations are used to calculate the estimation results. The sensitivity is then evaluated by taking the average over the 30 random samples. Table 4.11 reports the validation result for each scenario.

Table 4.11. Sensitivity analysis results of the multinomial case

Scenario	1					2				
Parameter	True Value	Mean	S.D.	MAPE	PMSE	True Value	Mean	S.D.	MAPE	PMSE
α_i	-0.6	-0.61	0.04	9.03	0.73	-0.6	-0.61	0.04	9.32	0.73
α_j	0.9	0.92	0.06	9.34	1.11	0.9	0.89	0.05	8.10	0.88
α_{ij}	-0.3	-0.30	0.03	6.69	0.21	-0.3	-0.30	0.03	6.85	0.25
$\beta_{i,1}$	-0.9	-0.90	0.04	2.68	0.10	-0.9	-0.89	0.04	2.96	0.13
$\beta_{j,1}$	0.6	0.60	0.03	4.83	0.26	0.6	0.59	0.03	4.72	0.24
$\beta_{ij,1}$	0.3	0.30	0.04	8.54	0.37	0.3	0.30	0.04	9.41	0.38
$\beta_{i,2}$	0.6	0.61	0.03	4.77	0.20	0.6	0.61	0.03	3.12	0.11
$\beta_{j,2}$	0.3	0.31	0.03	8.33	0.41	0.3	0.30	0.03	8.62	0.34
$\beta_{ij,2}$	-0.9	-0.90	0.03	3.93	0.17	-0.9	-0.90	0.03	3.05	0.12
σ_{11}	1.5	1.49	0.06	2.82	0.22	1.5	1.51	0.06	3.15	0.22
σ_{22}	1.2	1.21	0.05	3.16	0.17	1.2	1.19	0.05	3.18	0.15
σ_{12}	0.3	0.30	0.04	11.43	0.59	-0.1	-0.11	0.04	32.30	1.73
σ_{13}	0.2	0.19	0.06	26.57	2.28	-0.2	-0.19	0.06	23.92	2.03
σ_{23}	0.1	0.10	0.06	43.27	2.68	-0.3	-0.29	0.05	14.28	0.92

Scenario	3					4				
Parameter	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**
α_i	-0.6	-0.61	0.04	6.58	0.45	-0.6	-0.59	0.04	8.11	0.73
α_j	0.9	0.95	0.06	10.61	1.89	0.9	0.91	0.06	11.07	1.85
α_{ij}	-0.3	-0.31	0.03	7.62	0.29	-0.3	-0.30	0.03	7.00	0.24
$\beta_{i,1}$	-0.9	-0.90	0.04	2.98	0.13	-0.9	-0.88	0.03	2.62	0.11
$\beta_{j,1}$	0.6	0.61	0.04	5.28	0.27	0.6	0.61	0.04	5.00	0.20
$\beta_{ij,1}$	0.3	0.30	0.04	9.37	0.42	0.3	0.31	0.04	11.69	0.54
$\beta_{i,2}$	0.6	0.60	0.02	2.59	0.06	0.6	0.60	0.02	2.56	0.05
$\beta_{j,2}$	0.3	0.30	0.02	5.26	0.13	0.3	0.31	0.02	5.96	0.19
$\beta_{ij,2}$	-0.9	-0.90	0.02	2.30	0.07	-0.9	-0.90	0.02	1.81	0.05
σ_{11}	1.5	1.48	0.06	2.95	0.19	1.5	1.49	0.06	2.91	0.19
σ_{22}	0.6	0.61	0.02	3.03	0.10	0.6	0.60	0.03	3.76	0.11
σ_{12}	0.3	0.30	0.03	6.79	0.21	-0.1	-0.10	0.03	25.03	0.81
σ_{13}	0.2	0.20	0.06	27.05	2.04	-0.2	-0.18	0.06	20.69	1.47
σ_{23}	0.1	0.10	0.04	31.44	1.80	-0.3	-0.29	0.04	9.50	0.43

Scenario	5					6				
Parameter	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**
α_i	-0.6	-0.61	0.04	7.37	0.55	-0.6	-0.63	0.05	8.65	0.86
α_j	0.9	0.92	0.06	10.33	1.60	0.9	0.90	0.05	7.92	0.84
α_{ij}	-0.3	-0.30	0.03	7.80	0.32	-0.3	-0.30	0.03	7.06	0.28
$\beta_{i,1}$	-0.9	-0.90	0.03	2.28	0.07	-0.9	-0.90	0.03	2.60	0.10
$\beta_{j,1}$	0.6	0.60	0.03	2.90	0.08	0.6	0.60	0.03	3.19	0.10
$\beta_{ij,1}$	0.3	0.30	0.03	7.58	0.25	0.3	0.30	0.03	5.86	0.16
$\beta_{i,2}$	0.6	0.60	0.02	3.08	0.08	0.6	0.60	0.02	2.14	0.04
$\beta_{j,2}$	0.3	0.29	0.02	5.79	0.16	0.3	0.30	0.02	6.17	0.16
$\beta_{ij,2}$	-0.9	-0.90	0.02	2.17	0.07	-0.9	-0.90	0.02	1.55	0.03
σ_{11}	0.8	0.82	0.03	3.69	0.14	0.8	0.80	0.03	3.63	0.13
σ_{22}	0.6	0.60	0.02	3.41	0.10	0.6	0.59	0.03	3.51	0.10
σ_{12}	0.3	0.31	0.02	7.96	0.27	-0.1	-0.10	0.02	17.69	0.53
σ_{13}	0.2	0.21	0.05	14.55	0.78	-0.2	-0.20	0.04	20.08	1.43
σ_{23}	0.1	0.10	0.04	33.47	1.63	-0.3	-0.29	0.04	10.33	0.51

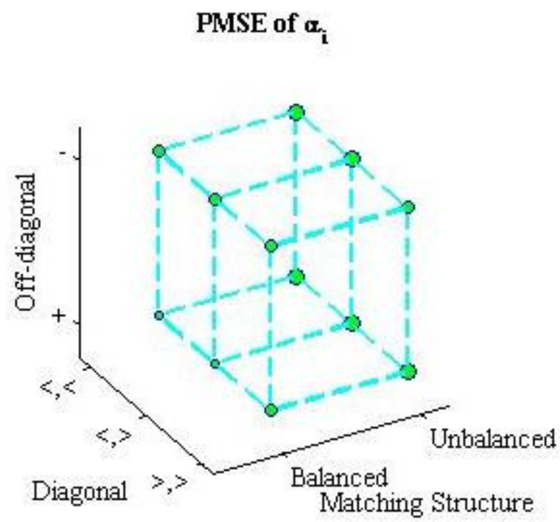
Scenario	7					8				
Parameter	True Value	Mean	S.D.	MAPE	PMSE	True Value	Mean	S.D.	MAPE	PMSE
α_i	-0.6	-0.55	0.07	18.09	2.73	-0.6	-0.60	0.07	11.42	1.42
α_j	0.9	1.02	0.09	18.68	4.46	0.9	0.98	0.10	13.28	2.46
α_{ij}	-0.3	-0.31	0.05	12.21	0.77	-0.3	-0.32	0.05	13.62	0.77
$\beta_{i,1}$	-0.9	-0.91	0.06	6.49	0.57	-0.9	-0.91	0.07	6.12	0.59
$\beta_{j,1}$	0.6	0.59	0.06	8.46	0.67	0.6	0.61	0.06	7.86	0.56
$\beta_{ij,1}$	0.3	0.32	0.06	14.25	0.96	0.3	0.29	0.06	16.55	1.11
$\beta_{i,2}$	0.6	0.60	0.06	7.63	0.56	0.6	0.59	0.06	9.97	1.01
$\beta_{j,2}$	0.3	0.29	0.05	16.30	1.20	0.3	0.33	0.05	15.05	1.15
$\beta_{ij,2}$	-0.9	-0.89	0.05	5.93	0.52	-0.9	-0.90	0.05	5.55	0.43
σ_{11}	1.5	1.50	0.10	4.35	0.46	1.5	1.52	0.10	5.14	0.72
σ_{22}	1.2	1.26	0.08	6.89	0.81	1.2	1.19	0.08	4.17	0.33
σ_{12}	0.3	0.32	0.07	16.89	1.30	-0.1	-0.10	0.06	53.98	4.20
σ_{13}	0.2	0.17	0.10	50.63	7.48	-0.2	-0.22	0.11	51.80	8.40
σ_{23}	0.1	0.12	0.10	86.22	10.31	-0.3	-0.30	0.10	28.36	4.20

Scenario	9					10				
Parameter	True Value	Mean	S.D.	MAPE	PMSE	True Value	Mean	S.D.	MAPE	PMSE
α_i	-0.6	-0.55	0.07	17.53	2.79	-0.6	-0.59	0.07	15.47	2.00
α_j	0.9	1.08	0.10	24.42	8.69	0.9	0.98	0.09	17.16	4.52
α_{ij}	-0.3	-0.32	4.92	12.22	0.79	-0.3	-0.30	0.05	13.75	0.87
$\beta_{i,1}$	-0.9	-0.90	0.06	5.21	0.39	-0.9	-0.90	0.06	5.90	0.53
$\beta_{j,1}$	0.6	0.59	0.06	7.96	0.51	0.6	0.61	0.06	7.31	0.52
$\beta_{ij,1}$	0.3	0.30	0.06	16.32	1.13	0.3	0.30	0.06	16.18	1.06
$\beta_{i,2}$	0.6	0.60	0.04	4.70	0.22	0.6	0.60	0.04	5.38	0.27
$\beta_{j,2}$	0.3	0.31	0.04	10.44	0.51	0.3	0.31	0.04	10.28	0.54
$\beta_{ij,2}$	-0.9	-0.90	0.04	2.99	0.13	-0.9	-0.91	0.04	3.20	0.14
σ_{11}	1.5	1.54	0.10	5.80	0.76	1.5	1.51	0.10	5.43	0.88
σ_{22}	0.6	0.63	0.04	6.01	0.28	0.6	0.61	0.04	6.74	0.35
σ_{12}	0.3	0.31	0.05	10.50	0.49	-0.1	-0.11	0.05	37.34	2.36
σ_{13}	0.2	0.22	0.11	48.34	6.98	-0.2	-0.20	0.10	44.91	6.92
σ_{23}	0.1	0.11	0.07	62.33	6.01	-0.3	-0.29	0.06	20.77	2.10

Scenario	11					12				
Parameter	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**	**True Value**	**Mean**	**S.D.**	**MAPE**	**PMSE**
α_i	-0.6	-0.54	0.07	13.68	2.02	-0.6	-0.56	0.06	16.28	3.03
α_j	0.9	1.09	0.11	23.99	9.48	0.9	1.05	0.09	24.22	10.01
α_{ij}	-0.3	-0.31	0.05	13.39	0.97	-0.3	-0.30	0.05	15.52	0.90
$\beta_{i,1}$	-0.9	-0.89	0.04	3.35	0.16	-0.9	-0.91	0.04	4.33	0.24
$\beta_{j,1}$	0.6	0.60	0.04	6.37	0.37	0.6	0.59	0.04	5.46	0.27
$\beta_{ij,1}$	0.3	0.31	0.04	10.78	0.47	0.3	0.31	0.04	11.72	0.60
$\beta_{i,2}$	0.6	0.60	0.04	4.74	0.23	0.6	0.61	0.04	4.20	0.18
$\beta_{j,2}$	0.3	0.30	0.04	10.10	0.45	0.3	0.32	0.04	9.10	0.42
$\beta_{ij,2}$	-0.9	-0.90	0.04	2.47	0.09	-0.9	-0.90	0.03	2.89	0.12
σ_{11}	0.8	0.80	0.05	4.44	0.24	0.8	0.81	0.05	5.57	0.34
σ_{22}	0.6	0.61	0.04	4.76	0.25	0.6	0.60	0.04	5.08	0.25
σ_{12}	0.3	0.30	0.04	8.68	0.31	-0.1	-0.10	0.03	22.73	0.88
σ_{13}	0.2	0.20	0.07	34.19	3.49	-0.2	-0.21	0.07	24.49	1.94
σ_{23}	0.1	0.08	0.07	74.04	7.25	-0.3	-0.28	0.06	13.29	0.73

Results show that the estimated parameters are close to their true values generally. The difference between the estimated values and the pre-defined values is less than 0.02 in most cases. Such a small difference indicates that the proposed estimation method yields an excellent parameter recovery capability and validates the proposed model with the multinomial outcome.

Figure 4.6 further uses 3-D scatter plots to show PMSE for all parameters across scenarios.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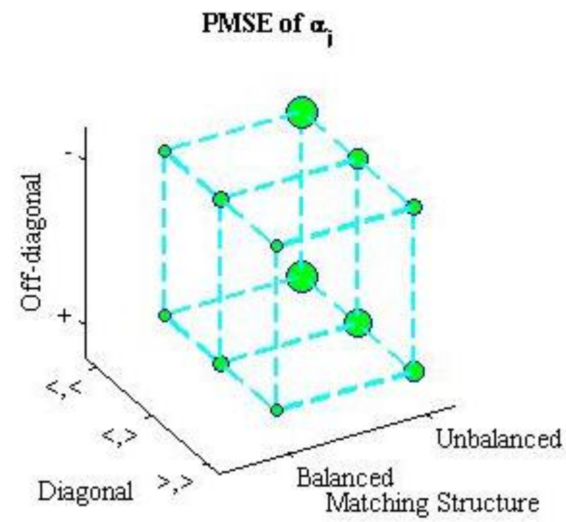
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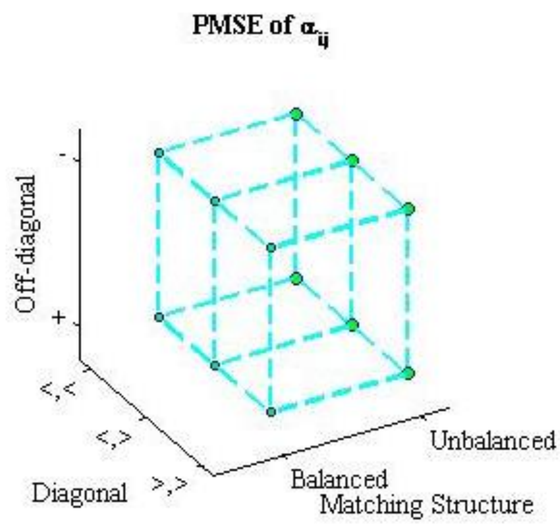
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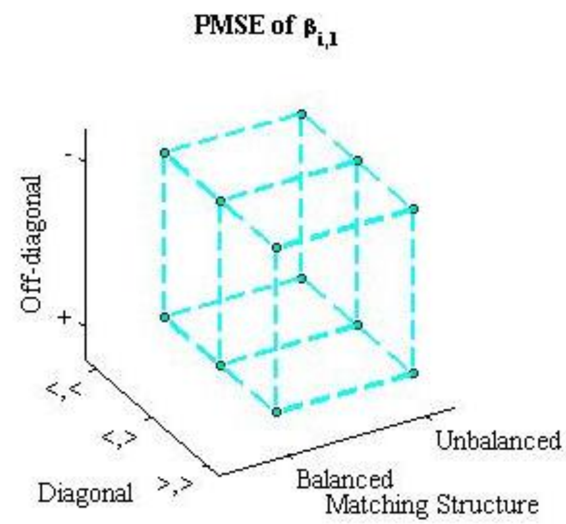
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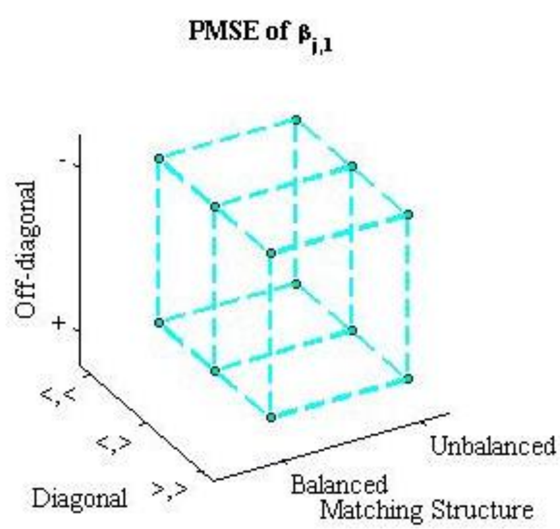
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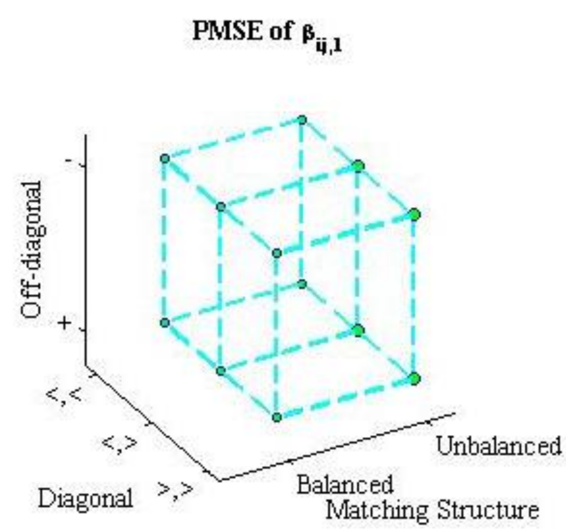
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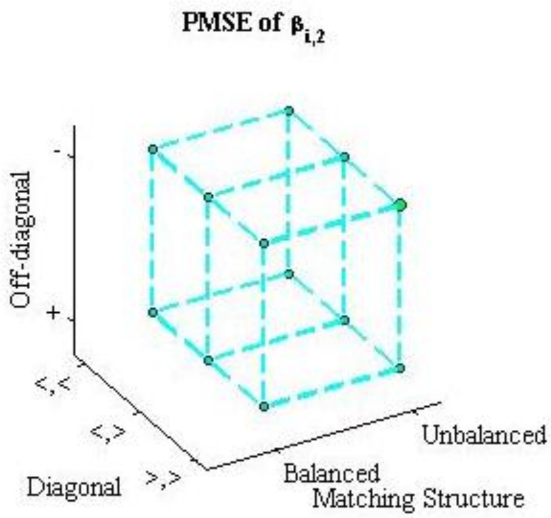
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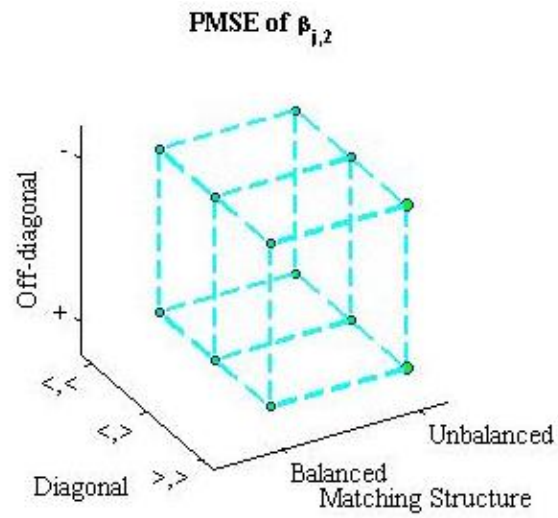
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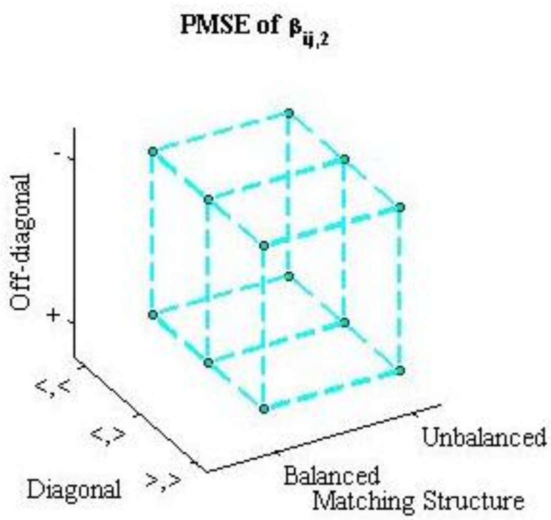
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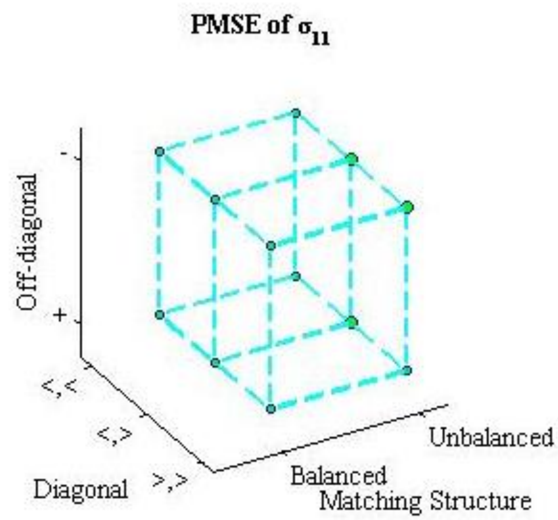
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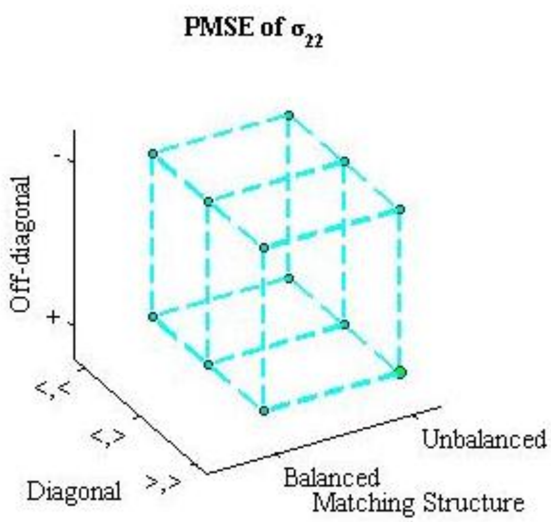
Matching Structure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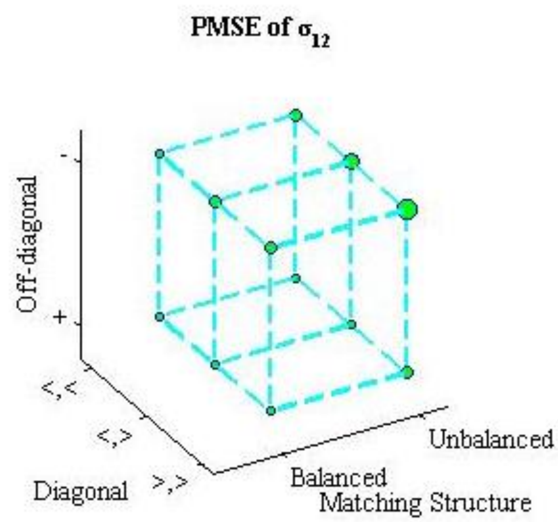
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Matching Structure

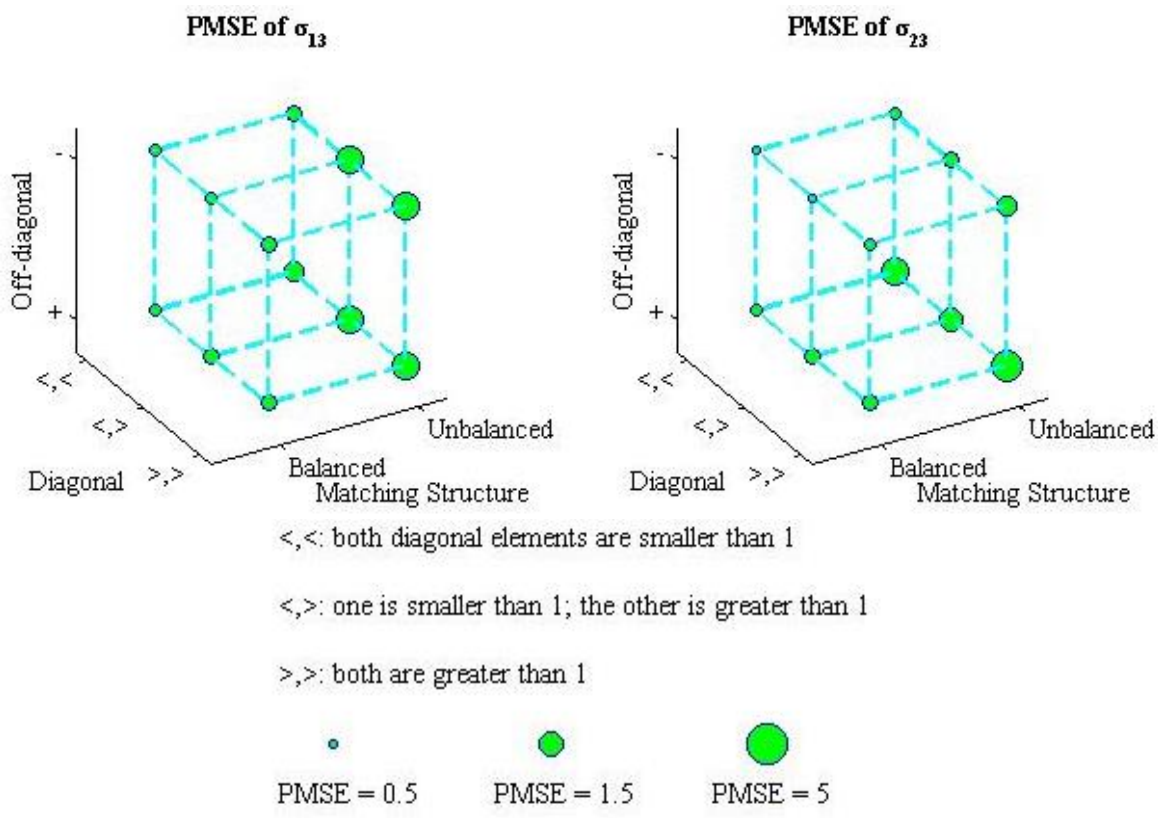

Figure 4.6. PMSE of 30 random samples in the multinomial case

Evidence also shows variation over scenarios. For example, balanced matching structures have a better parameter recovery compared to unbalanced matching structures. Such an advantage can be observed significantly in the estimation result of α in the matching equation. The reason is that attributes of agent-specific factors (e.g., w_i and w_j) vary less in unbalanced scenarios. Therefore, the estimated values are more dependent on the values of the randomly generated w_i and w_j, yielding higher PMSE. This finding is also supported by the estimated result of γ where the parameter of pair-specific factors presents similar recovery in balanced matching scenarios and unbalanced scenarios. In addition, the sample size slightly affects the estimation accuracy.

Given the good parameter recovery capability, the proposed models are successfully validated. The validation indicates the proposed models can be used in empirical datasets, which shows the contribution of this dissertation.

5. Model Application I: Airline-Airport Collaboration

With the specified and validated models, this chapter applies the proposed collaborative decision making model with ordinal outcomes to analyze flight on-time performance with the consideration of airline-airport collaboration.

5.1 Introduction to Airline-Airport Collaboration

The passage of the Airline Deregulation Act in 1978 gave U.S. airlines almost full freedom to determine which airports to serve. In recent years, airports are also experiencing privatization to operate like business entities, in which selecting the airline partner is key to the business. As a result, the airline-airport collaboration is formed to benefit both agents' businesses. Such collaboration has brought positive outcomes: high profits, low airfare, and local economic development. It also raises concerns about anti-competitive consequences (e.g., an airline's dominance is strengthened at an airport to gain competitive advantages over other airlines) and poor service performance of non-signatory airlines (e.g., airlines who do not use the airport as a hub). To understand the formation and joint response of the airline-airport collaboration, historical and regional factors are always attributed, which exhibit high heterogeneity and are incapable of being analyzed by a general model. However, the questions can be also understood by some common strategies from the perspective of business development. This dissertation uses the proposed joint response model to analyze the casual relationship between a set of business development factors and the formation/outcome of airline-airport collaboration in the U.S.

Airlines have incentives to collaborate with airports to implement their operations. For major airlines, most of them operate in a hub-and-spoke network: Hubs are airports that an airline uses as a transfer point to get passengers to their final destinations; spokes are air routes that connect non-hub airports to hub airports. Without a reliable support from airports, especially hub airports, airlines could not operate their businesses in an efficient way. Many strong collaborative relationship examples are observed at hub airports in the U.S. For example, JetBlue Airways financially invested in the John F. Kennedy International Airport (JFK) terminal 5 to gain direct control for 30 years. For airlines operating in a point-to-point network, airlines' dependence on airports is not as strong as airlines operating in hub-and-spoke networks, but airlines still consider the cost and benefit that airports could provide, such as various fees/charges for using the airport, potential passenger volume, and locations of airports. Collectively, the selection of serving airports for an airline is determined by the service provided by airports, relative geographic relationship with existing network, and potential markets.

Airports have started treating themselves as business entities due to the reduction in government supports. Although they are not fully independent to government subsidies, airport managers have to propose strategies to increase revenues and reduce costs from the perspective of free markets. One of the key strategies is to form a long-term alliance with airlines. With the alliance, airports could ask the airlines to cover part of the airport's expenses, lower administrative costs, and be competitive to other airports. Therefore, selecting a good airline collaborator is particularly important for airports to achieve their business objectives. As all airlines are potential collaborators of airports, airports could evaluate all of them and make the selection decision based on the evaluation. Key factors in the evaluation include airlines' service coverage, airline companies' sizes, and locations of current markets.

The airline-airport collaboration is a two-sided matching structure and exhibits complex underlying relationships. Per Fu et al. (2011), relationships could be categorized into the following types: (1) signatory airlines of airports, (2) airline ownership or control of airport facility, (3) long-term use contracts, (4) airport issuance of revenue bonds to airlines, (5) revenue sharing between airports and airlines, and (6) other relationships. No matter which type a relationship belongs to, an airline-airport relationship is considered as a collaborative relationship in this dissertation if any flight of an airline lands in the airport. With the observed relationship data, the proposed model is able to identify parameters that explain the effects of various factors on the formation of airline-airport collaboration.

Flight on-time performance is a crucial component of airline/airport profits and customer satisfaction. IATA creates standardized codes for airlines to report the reasons for flight departure delays. Delays codes consist of the following categories: passenger and baggage, cargo and mail, aircraft and ramp handling, technical and aircraft equipment, damage to aircraft, automated equipment failure, flight operations and crewing, weather, air traffic flow management restrictions, airport and government authorities, reactionary, miscellaneous, and other reasons (Boone, 2009). Classifying the reasons would lead to the understanding of airlines' and airports' characteristics, and their collaboration. For example, an airline and its hub airport that have a deep collaborative relationship are more likely associated with fast ground preparation, ideal flight schedules, and other conveniences, resulting in low delay possibility. In contrast, airports and airlines that do not have a deep collaborative relationship are more likely to encounter miscommunication and slow baggage preparation, and consequently, flight delays. Existing studies miss the perspective of airline-airport collaboration, which leads to an insufficient understanding of flight delay issues, thus resulting in ineffective strategies of reducing flight delays. Therefore, this dissertation fills the void by investigating the effect of airline-airport collaboration on flight on-time performance.

5.2 Data Description

This dissertation uses a set of datasets from the Bureau of Transportation Statistics to analyze flight on-time performance considering various influential factors as well as airline-airport collaboration. Two regression equations are specified, with the first as the matching equation and the second as the joint decision making equation with ordinal outcomes. The matching equation disentangles how and why the investigated airlines are matched with the investigated airports. The joint decision making equation analyzes how airlines and airports jointly determine flight on-time performance.

The Airline On-Time Performance Data (U.S. Department of Transportation, 2015) contains the on-time performance of domestic flights in 2014 for 14 major U.S. airlines at 324 major U.S. airports. The description of the investigated airlines is listed in Table 5.1. Note that some airlines report their data jointly after their merge announcement (e.g., Delta and Northwest). Other airlines still report separately in 2014 even though the companies have been merged (e.g., Southwest and AirTran). Figure 5.1 shows the locations of the 324 major airports, which spread over the continent, Alaska, Hawaii, and other major territories. This dataset includes information of not only the on-time performance, but also the paired relationship between airlines and airports. With such an observed relationship data, the matching equation can be specified to identify the parameters of independent variables.

Table 5.1. Airline description

Airline ID	Airline Code	Airline Description
1	AS	Alaska Airlines Inc.
2	AA	American Airlines Inc.
3	MQ	Envoy Air
4	DL	Delta Air Lines Inc.
5	EV	ExpressJet Airlines Inc.
6	F9	Frontier Airlines Inc.
7	HA	Hawaiian Airlines Inc.
8	B6	JetBlue Airways
9	OO	SkyWest Airlines Inc.
10	WN	Southwest Airlines Co.
11	FL	AirTran Airways Corporation
12	UA	United Air Lines Inc.
13	US	US Airways. Inc.
14	VX	Virgin America

Figure 5.1. Locations of the 324 airports

The matching equation contains three independent variables, which are airlines' current assets on their financial statements (U.S. Department of Transportation, 2015), airports' annual passenger volume (U.S. Department of Transportation, 2015), and the distance from an airport to the closest airline's hub (U.S. Department of Transportation, 2015). Current assets reflect the size of an airline company, and passenger volume capture the size of an airport. These two variables could explore the attractiveness of each side in terms of their scales. The distance from an airport to the closest airline's hub measures relative geographic proximity between two sides. Collectively, the three independent variables capture the characteristics of airlines and airports, and their joint characteristics. Note that other variables can be also added into the matching equation to improve the model's explanatory power. However, as an illustrative application, this dissertation considers only the mentioned variables.

The joint decision making equation is assumed to have flight on-time performance as the ordinal outcome. In the Airline On-Time Performance Data (U.S. Department of Transportation, 2015), on-time performance is defined as the percentage of on-time departure (e.g., flights depart within 15 minutes of the scheduled time). Such percentage numbers are further coded as an ordinal variable with three categories as shown in Table 5.2. In this particular empirical study, the ordinal code can better serve for the analysis than the percentage numbers because some on-time performance data has extreme values, such as a 20% flight on-time performance for Frontier at Sioux Falls Regional Airport. These extreme values may result from misreporting or from the number of flights for an airline at an airport being too small.

Table 5.2. Ordinal outcome of flight on-time performance

Percentage of On-time Departure	Ordinal Code	Number of Observations
<70	1	349
Between 70 and 85	2	763
>85	3	179

As this application serves as an illustration of the proposed model, only three variables are used as explanatory variables to capture a subset of the delay reasons. The three variables are the airline's load factor (U.S. Department of Transportation, 2015), airport's latitude, and if the airport is the airline's hub. These three variables are able to partly capture the effects of weather and flight operation on flight delays from the airline's side, airport's side, and the interaction of airline and airport.

A summary of the variables in the model is reported in Table 5.3.

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Table 5.3. Summary statistics of variables in the proposed model

Variable Name	Definition	Size	Mean	S.D.	Min	Max
Matching Equation						
I	Number of airlines	14				
J	Number of airports	324				
ASST	Logarithm of airlines' current assets in USD	4536	6.75	0.67	5.73	7.75
VOLM	Logarithm of airports' annual passenger volume in persons	4536	5.19	0.94	2.93	2.93
DIST	Distance from an airport to an airline's hub in thousand miles	4536	0.94	1.15	0	7.35
Joint Decision Making Equation with Ordinal Outcome						
DELAY	Level of delay (refer to Table 5.2)					
LDFCT	Airline's load factor	1291	0.84	0.03	0.77	0.90
LAT	Airport's latitude in degree/100 north	1291	0.37	0.07	-0.14	0.71
HUB	Binary variable: 1 if the airport is the airline's hub; 0 if not	1291	0.07	0.25	0	1

5.3 Results Analysis

The flight on-time performance data are analyzed using the proposed collaborative decision making model. The estimation process runs 15,000 iterations with the first 10,000 iterations as the "burn-in" period. The estimation results are obtained from simulated values in the last 5,000 iterations. The traces of estimated parameters are presented in Figure 5.2.

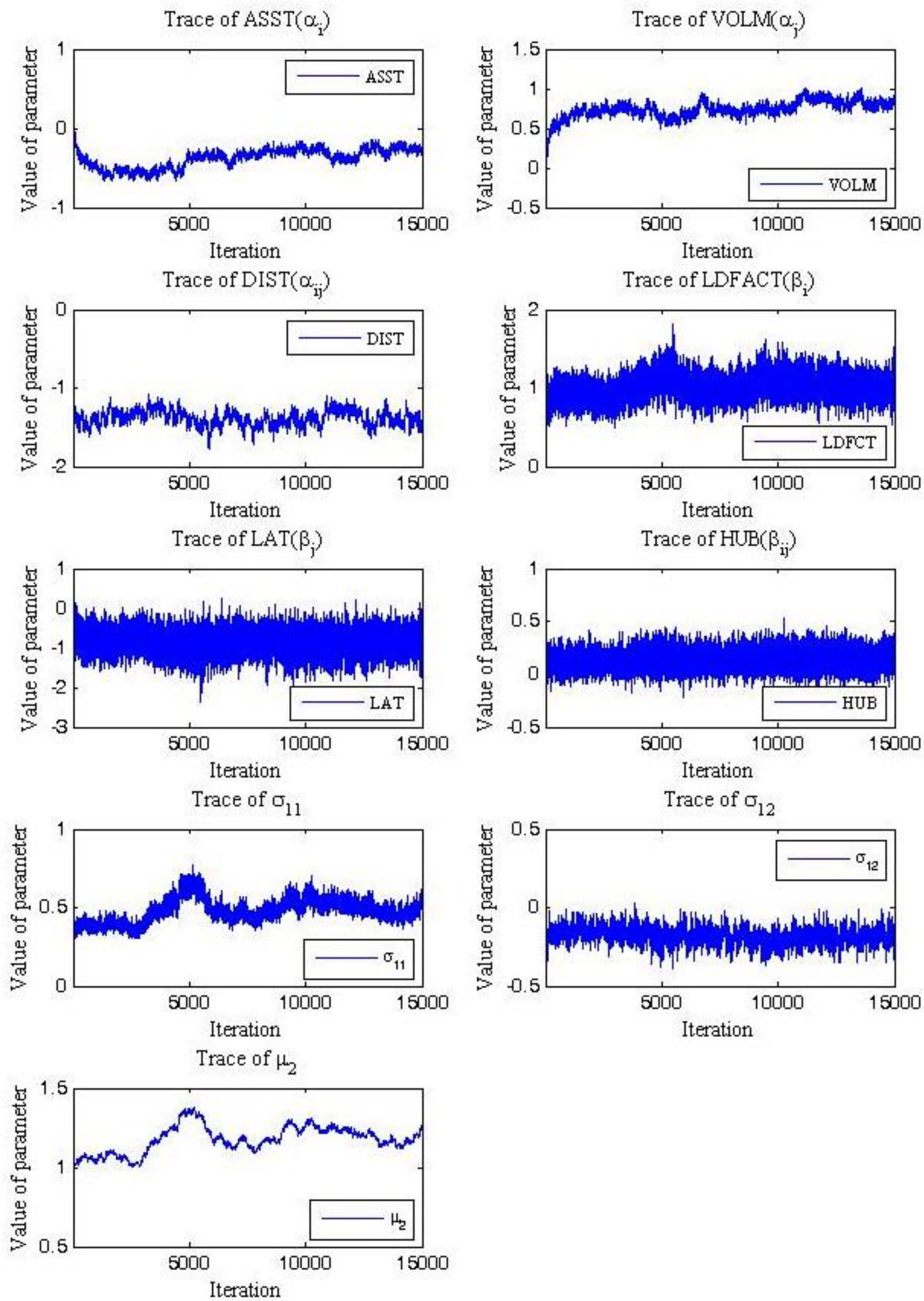


Figure 5.2. Traces of parameters in the airline-airport collaboration

The traces of all estimated parameters are stable after the first 10,000 iterations. Therefore, the last 5,000 iterations can be used to derive the posterior distributions. The posterior distributions of estimated parameters are shown in Figure 5.3.

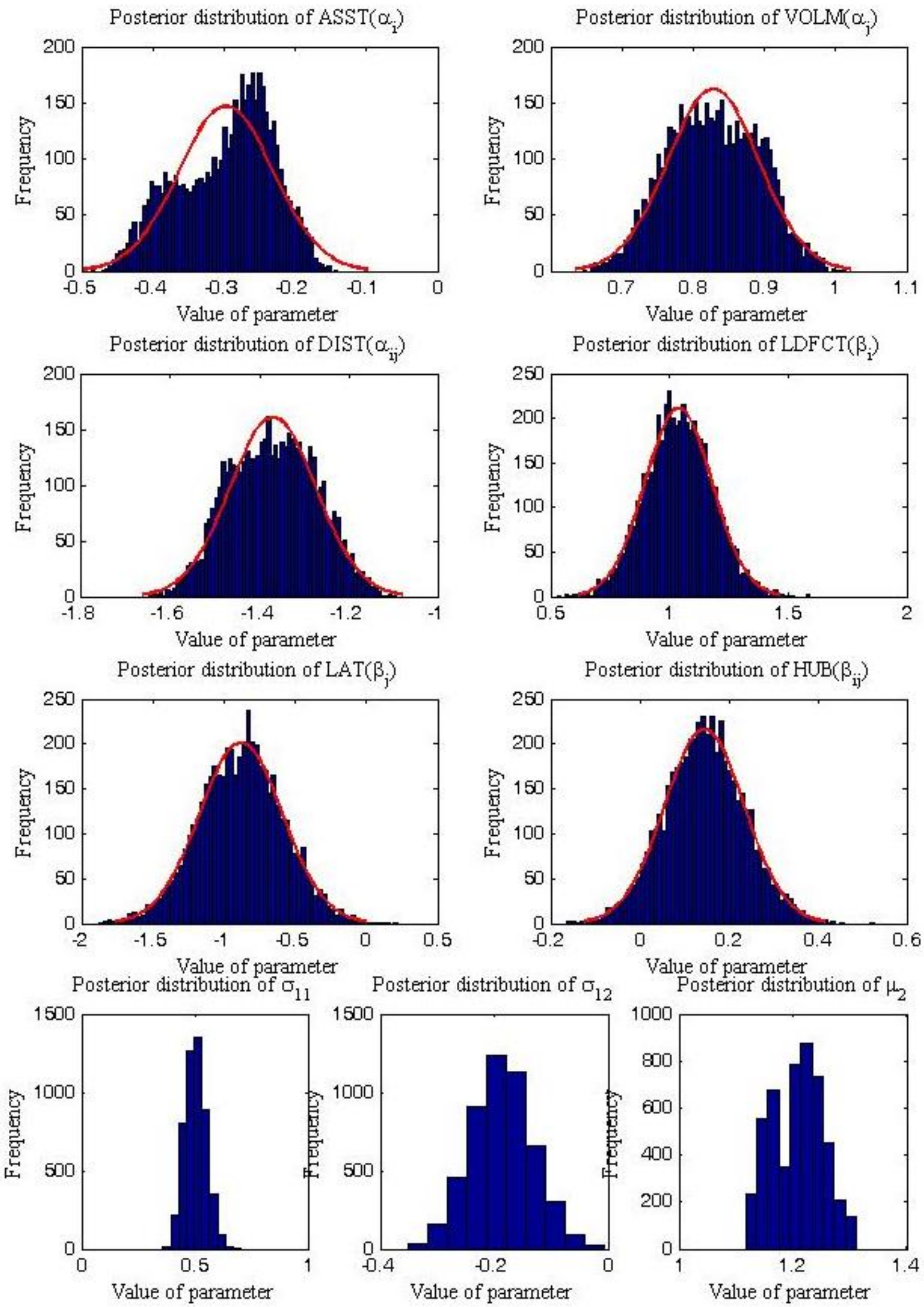


Figure 5.3. Posterior distributions of estimated parameters in the airline-airport collaboration

A summary of the estimation results is reported in Table 5.4.

Table 5.4. Estimation results of airline-airport collaboration

Parameters		Mean	S.D.	Pseudo t-stat	95% CI low	95% CI up
α_i	ASST	-0.297	0.066	-4.50	-0.426	-0.168
α_j	VOLM	0.828	0.064	12.93	0.703	0.953
α_{ij}	DIST	-1.370	0.097	-14.12	-1.560	-1.180
β_i	LDFCT	1.039	0.141	7.37	0.763	1.315
β_j	LAT	-0.882	0.293	-3.01	-1.456	-0.308
β_{ij}	HUB	0.145	0.090	1.61	-0.031	0.321
σ_{11}		0.503	0.046			
σ_{12}		-0.187	0.055			
μ_1		0 (fixed)				
μ_2		1.208	0.044			
Number of matched pairs		1291				
Number of unmatched pairs		3245				
Log likelihood at null		-8014				
Log likelihood		-7742				
Likelihood ratio		543.46	0.000			

The likelihood ratio test shows that the fitted model significantly improves model's goodness-of-fit. According to the estimated posterior distributions and means/standard deviations, all estimated parameters are significantly different from zero. Such a conclusion is also supported by the pseudo t-statistics. Except parameter θ, all other parameters are significant at the 0.05 level. Parameter θ is significant at about the 0.1 level. These estimated parameters reveal interesting and important implications about flight on-time performance.

In the matching equation, the airline's asset has a negative coefficient on pairwise utility, indicating that small airline companies are more attractive to airports than large ones. Small airline companies are usually associated with low airfare, which could attract more passengers to the airports. A good number of passengers bring the airport service charge earnings, prosperous businesses, and flourishing investments. These stimulated incomes are consistent with airports' business objectives, increasing revenue and keeping them competitive with other airports. Airports' passenger volume has a positive coefficient on pairwise utility, indicating that large airports are more attractive to airlines. Passengers normally use a large airport as a transfer point to their final destination. An airline could to serve a segment for these passengers with transfer in their trips. Besides, large airports normally provide better service and are close to concentrated markets. All of these features may bring airlines more passengers and thus high revenue. The distance from an airport to an airline's hub has a negative coefficient, indicating that airlines and airports are likely paired up at airports that are near to airlines' main markets. Most airlines operate in a hub-and-spoke network. Hubs are the main markets of airlines. Airports near to hubs are connected by short-haul flights, and passengers could reach other airports by transferring at hubs. Therefore, serving these airports could improve the number of airlines' customers. In practice, targeting these airports is a cost efficient strategy for airlines operating in hub-and-spoke networks. For point-to-point service networks, a landed airport is also expected to be near to the airline's current network. Airlines at faraway airports have to face concerns with brand awareness, reliability, and operation cost, reducing the attractiveness to airlines.

In the joint decision making decision equation, the airline's load factor has a positive coefficient. Airline companies with high load factors are associated with good operation management, such as fleet assignment, scheduling, and demand forecast. These airlines with good operation conditions are favored by airports. An airport's latitude is an indicator of extreme weather: airports at higher latitudes are more likely to suffer from heavy snow, which could delay flights' operations. A negative coefficient is consistent with such an expectation. A positive coefficient of hubs indicates an airline at its hub airport is more likely to catch up with the schedule. Airlines at hub airports may have prioritized flight schedules, flexible boarding gate assignment, and smooth communication with airports. Such a hub effect confirms that the airline-airport collaboration has a direct effect on flight on-time performance.

Airline-airport collaboration also has indirect effects on flight on-time performance through the correlation of the two equations. The two free variables in the variance-covariance matrix are significantly different from zero. The off-diagonal element σ_{12} is -0.187, indicating that the matching equation and the joint decision making equation are negatively related to each other. The uncaptured effects in establishing collaboration would conversely impact the on-time performance. For example, well-known airlines may be associated with attractiveness, but these airlines are not good at operating on schedule. Coupled with the estimated diagonal element σ_{11}, the conditional mean and variance of each equation can be obtained. In other words, without considering the correlation, parameter estimation would be biased. The estimated variance-covariance term confirms the necessity of considering the sample selection process in this airline-airport collaboration analysis.

In summary, the proposed model fits the airline-airport collaboration data well. The considerations of the intricate matching structure, mutual selection, and joint decision making significantly improve the model's goodness-of-fit and provide additional insights into the flight on-time performance problem. A series of explanatory variables are found useful in explaining the formation of airline-airport collaboration and outcome of flight on-time performance. The analysis results improve the understanding of these problems and may help to propose effective policies to reduce flight delay. This application shows the significance of how the proposed model contributes to empirical analyses.

6. Model Application II: Freight Carrier-Receiver Collaboration

This chapter uses the proposed joint response model with multinomial outcomes to analyze freight agents' responses to toll increases.

6.1 Introduction to Freight Agents' Interaction

The freight system transports a variety of supplies for modern life, generating tremendous benefits as well as negative impacts on quality of life, sustainability, and the environment. MAP-21 (Moving Ahead for Progress in the 21st Century) acknowledges the importance of freight research to support policy making (U.S. Department of Transportation, 2013) and calls for the design of effective freight policies to reduce congestion, mitigate pollution, and improve supply chain efficiency.

Understanding the behavior of freight agents who are the targets of a given policy, specifically how the agent would react to the policy, is essential to ensure that the policy has the desired effects. Unlike its passenger counterpart, where trip decisions are made solely by travelers, the response of logistics is determined by multiple agents, such as suppliers, carriers, and receivers. Disregarding interactions among these agents may prevent the research community from fully understanding the decision mechanism, leading to misleading assessments of policy effects on each individual decision maker and consequently, poor predictive power. Existing studies have investigated this issue, although the number of studies is limited. Holguin-Veras et al. (2015) argues that the freight activity is a result of the economic agents' interactions and these interactions determine the supply chain's response to freight policies. As they claimed, a number of agents are involved in the freight supply chain: suppliers take the role of producers/manufacturers and shippers, carriers conduct vehicular transportation of cargoes between shipping and receiving locations, and receivers are the recipients of cargoes. In freight decision-making processes, some decisions are made by the agents independently from the others, while other decisions are determined jointly by multiple agents. For example, decisions of delivery routes and trucking technologies are usually made by carriers solely. Delivery rates, sizes, and frequency, on the other hand, are impacted jointly by suppliers, carriers, and receivers. In typical cases, receivers have a greater influence on the other agents because receivers are the generators of freight demand and freight traffic.

Receivers are just like employers, who have high leverage on employees, and can mandate the working time and content. In a series of empirical studies reviewed in Holguin-Veras et al. (2015), carrier-centered policies tend to lead the carriers to enact behavioral responses because they have to obey the rules set by the receivers, and consequently, unintended policy results are found in some cases. On the other hand, receiver-centered policies are more effective because receivers are able to ensure the carriers change delivery patterns. A series of off-hour delivery studies (Holguín-Veras, Pérez, Cruz, & Polimeni, 2006; Holguín-Veras, Silas, Polimeni, & Cruz, 2008; Holguín-Veras et al., 2015) illustrate such effectiveness of receiver-centered policies: toll fee improvements may not move carriers to off-hours because receivers prefer regular business hours and they overpower carriers. Alternative policies that aim at receivers, such as financial incentives, offer the best way to switch delivery traffic to off-hours.

Therefore, understanding the interaction among freight agents and their relative market power in freight decision making is critical for the freight activities. In definition, the market power of a firm measures the ability to profitably raise the market price of a product or service. In this research, the market power of a freight agent characterizes the agent's ability to determine delivery price, time, frequency, and other freight-activity decisions. For example, if the receiver has more market power than the carrier, the receiver dominants decisions on freight-activity decisions. In addition, the market power of a specific agent (e.g. the receiver solely) may differ in different situations. For example, Holguin-Veras et al. (2008) found that receivers in the food and retail sectors are the most inclined to respond to policy change. Such a finding can be explained by their market power being higher than receivers in other industrial sectors.

On the basis of the existing research, this research enriches the existing literature by investigating market power of carriers by investigating multiple influential factors. A stated preference survey is conducted to collect data to investigate freight agents' behavior change in response to hypothetical toll increases. The change could transfer the increased toll to receivers and/or reduce delivery frequency. The ability to change behavior characterizes carriers' market power relative to receivers'. If the carriers have absolute power, they can transfer the increased tolls to receivers and keep the current delivery frequency without sacrificing any of their own profits. Otherwise, carriers have to absorb part or all of the increased tolls to avoid losing customers. Therefore, the degree to which the increased tolls are absorbed by carriers is a reasonable indicator of carriers' market power relative to receivers'. In order to obtain this degree of toll transfer, this study conducts a stated preference survey on the managers of carriers' companies in the New York State (NYS). Given hypothetical toll increases, the survey asks two questions:

1. Willingness of Transfer Question: If the toll increases by the selected amount, is your company likely to pass on any of the cost to customer? and
2. Willingness of Frequency Reduction Question: If the toll increases by the selected amount, is your company likely to reduce delivery frequency?

The survey also collects data about variables that could influence the behavior change to transfer toll increases, such as the types of commodity transported, the size of carriers' company, current toll fees, and carriers' typical delivery trips. On the basis of the information about typical delivery trips, variables that characterize carriers, receivers, and joint factors are selected. These factors are analyzed using the proposed joint response model to gain insights into carriers' relative market power.

6.2 Data Description

A stated preference survey was conducted to collect information on freight agents' response to hypothetical toll increases. The respondents are the managers of logistic companies in which carriers regularly use a tolled highway (at least once per month) in New York State (NYS). The survey first collects carriers' basic information, such as usage of the Electronic Toll Collection (ETC) system, fleet information, and delivery frequency. Then, information about carriers' typical delivery tours (i.e., the tour that the company serves most frequently) is collected including the origin and the final destination (most are located within NYS), the vehicle used, and the typical load size for the typical delivery tour.

The carriers are then given three hypothetical toll increase scenarios where the amount of toll increases are randomly selected from the following values: 10%, 20%, 40%, 80%, 100%, 120%, 160%, and 200%. In this dissertation, only one scenario is selected randomly for each respondent to avoid the complexity resulting from repeated choice effects. The carriers are asked to respond to the toll increases. Pre-defined possible responses include the toll transfer, frequency reduction, route change, and time-of-day of travel shift. This dissertation focuses on the first two possible responses because these two responses present the highest variances. The two possible responses lead to a multinomial outcome with four categories. Specifically, the four multinomial outcomes are

1. Transfer toll increase to customers and reduce delivery frequency
2. Transfer toll increase only
3. Reduce delivery frequency only
4. Neither transfer toll increase to customers nor reduce delivery frequency

The survey was conducted in May, 2014 and successfully collected 370 carriers' information. After filtering out missing values, 321 valid samples remain in the dataset with each sample representing one carrier. With the investigated highway exits of each carrier's typical tour, an origin-destination pair for each carrier can be derived. A map of exit locations is shown in Figure 6.1. The origins are perceived as the locations of carriers and the destinations are perceived as the locations of receivers. The destinations are further combined at the county level: if two destinations locate at the same county, they are treated as the same destination. The reason of combining nearby exits is that freight demand presents similar characteristics among nearby exits and can be analyzed conveniently by county-level characteristics. A list of counties that have highway exits is reported in Table 6.1. Therefore, a one-to-many matching structure is formed. The carrier side has 321 agents and each agent is paired with only one customer. The customer side has 22 agents and each agent is paired with multiple customers.

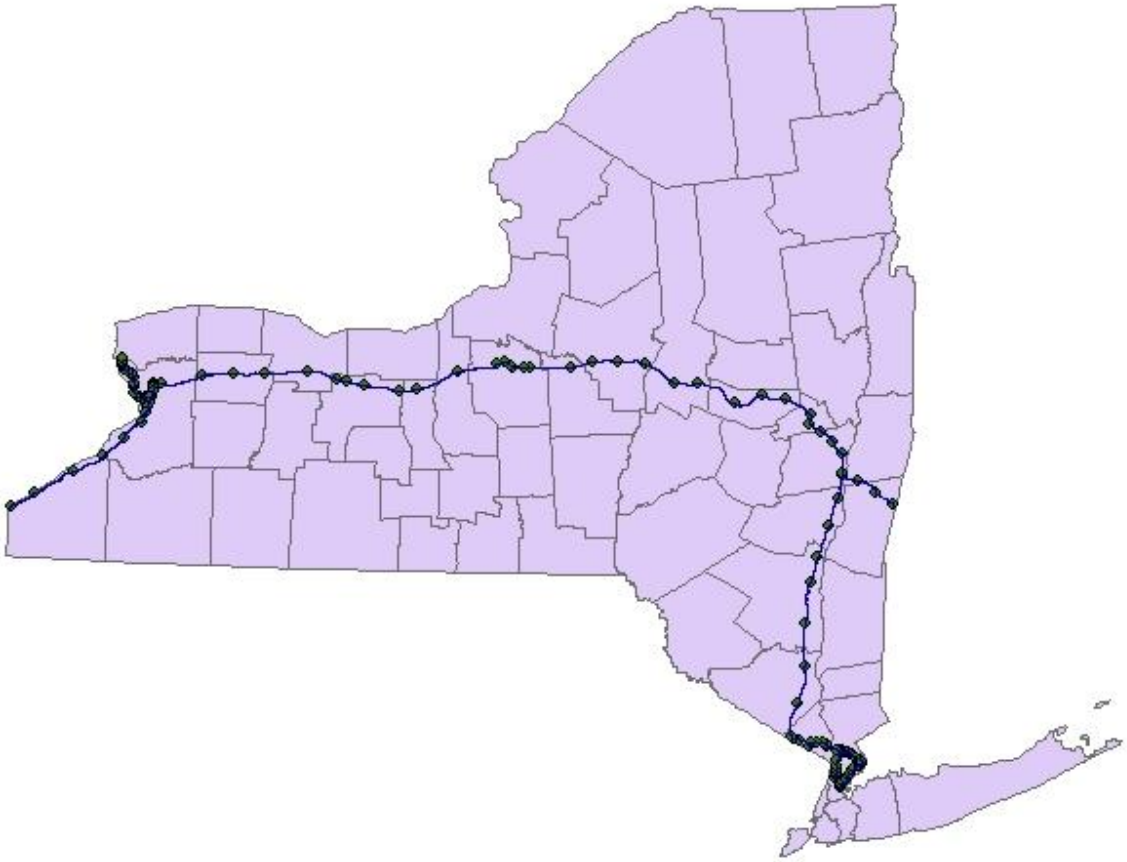


Figure 6.1. Locations of highway exits

Table 6.1. Important characteristics of counties that have highway exits

ID	County Name	Urban Population to Total Population	Logarithm of Median Household Income
1	Bronx	1.00	5.68
2	Westchester	0.97	5.54
3	Rockland	0.99	4.99
4	Orange	0.78	5.10
5	Ulster	0.54	4.84
6	Greene	0.27	4.26
7	Columbia	0.27	4.40
8	Albany	0.90	5.09
9	Rensselaer	0.69	4.81
10	Schenectady	0.92	4.76
11	Montgomery	0.59	4.29
12	Herkimer	0.48	4.42
13	Oneida	0.67	4.96
14	Madison	0.41	4.42
15	Onondaga	0.87	5.27
16	Cayuga	0.44	4.49
17	Seneca	0.41	4.13
18	Ontario	0.53	4.65
19	Monroe	0.94	5.48
20	Genesee	0.40	4.38
21	Erie	0.91	5.58
22	Chautauqua	0.56	4.73

The response to hypothetical toll increases and formation of the matching structure can be analyzed by a series of influential factors.

Table 6.2 summarizes the statistics of used variables in the joint response model.

Table 6.2. Summary statistics of variables in the proposed model

Variable Name	Definition	Size	Mean	S.D.	Min	Max
Matching Equation						
I	Number of carriers	321				
J	Number of customers	22				
FREQ	Logarithm of delivery frequency	321	0.92	0.51	0	4.00
UPOP	Urban population to total population at the county level	321	0.66	0.24	0.27	1.00
DIST	Distance between carrier's locations and customer's locations in hundred miles	321	1.69	1.18	0	4.94
Joint Response Equation						
MINC	logarithm of Household median income at county level	321	0.50	0.04	0.41	0.57
INCR	Hypothetical toll increase amount	321	0.96	0.63	0.1	2
SIZE	Logarithm of total number of vehicles. Alternative-specific variable for outcome 1	321	0.42	0.46	0	2.76
FARM	Binary variable: 1 if carrier is in farm sector; 0 if not. Alternative-specific variable for outcome 2	321	0.12	0.32	0	1
MEAT	Binary variable: 1 if carrier is in meat sector; 0 if not. Alternative-specific variable for outcome 3	321	0.03	0.18	0	1

6.3 Results Analysis

The freight survey data are analyzed using the proposed joint response model. The estimation process runs 700 iterations with the first 400 iterations as the “burn-in” period. The estimation results are obtained from simulated values in the last 300 iterations. The traces of estimated parameters are presented in Figure 6.2.

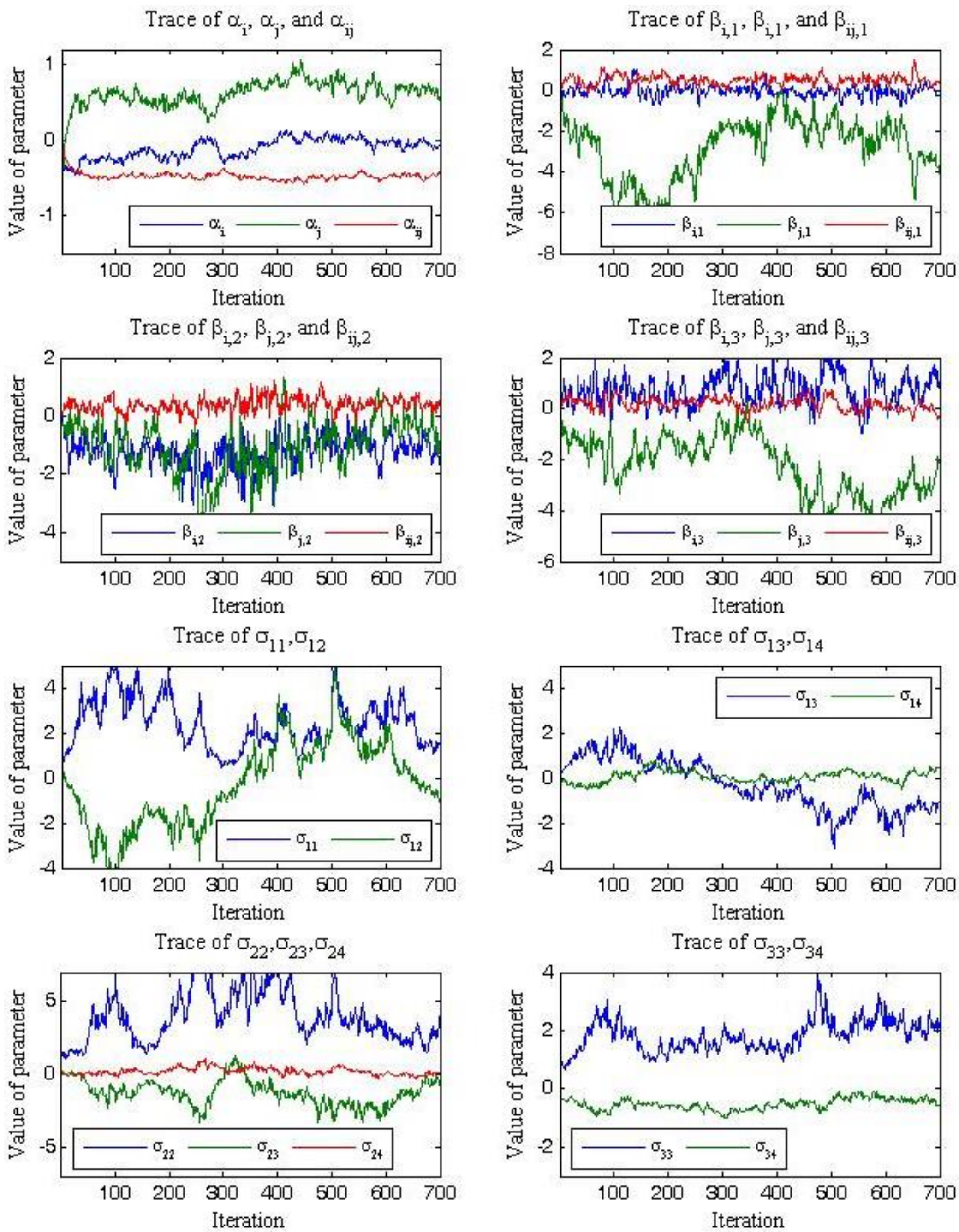
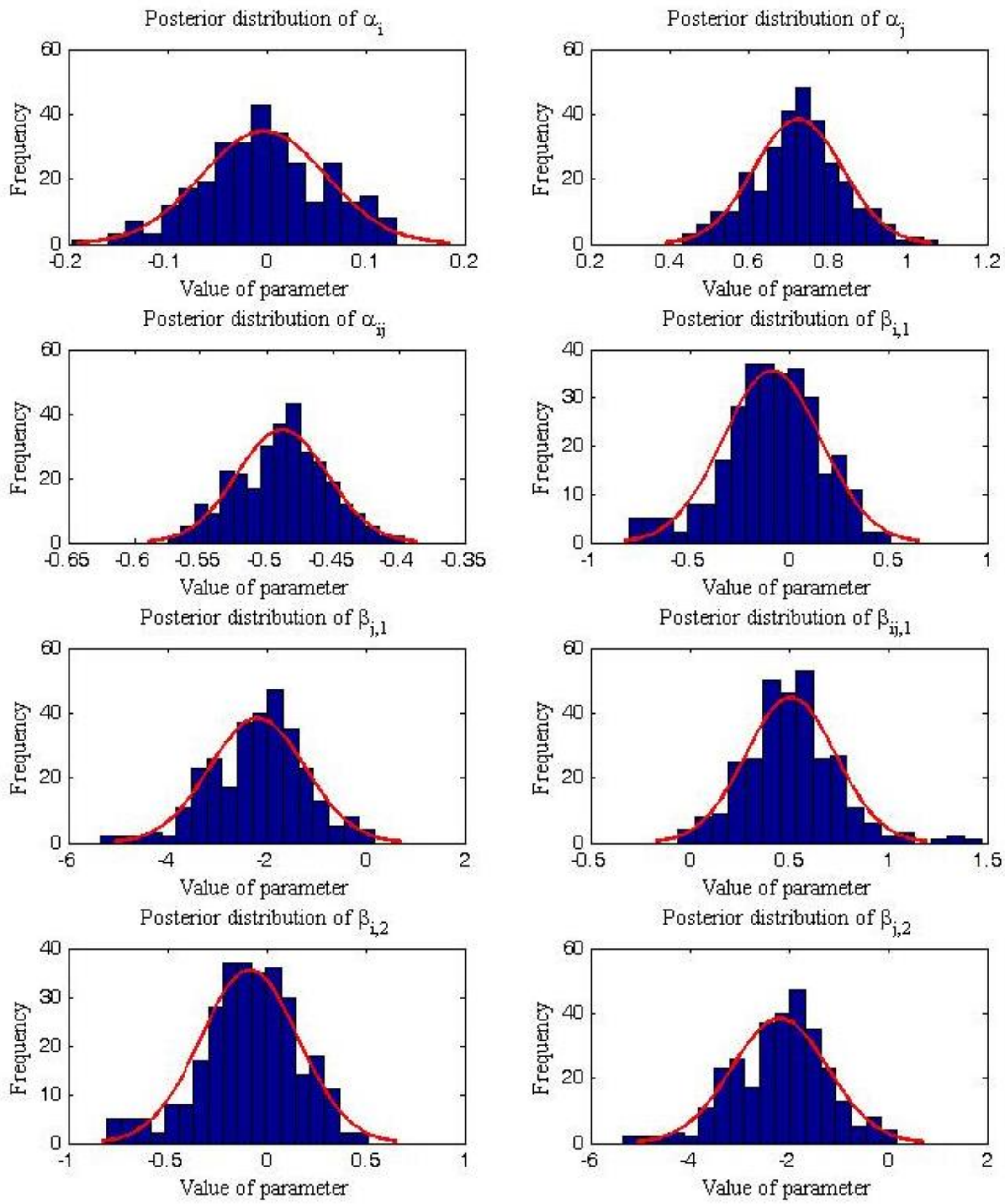
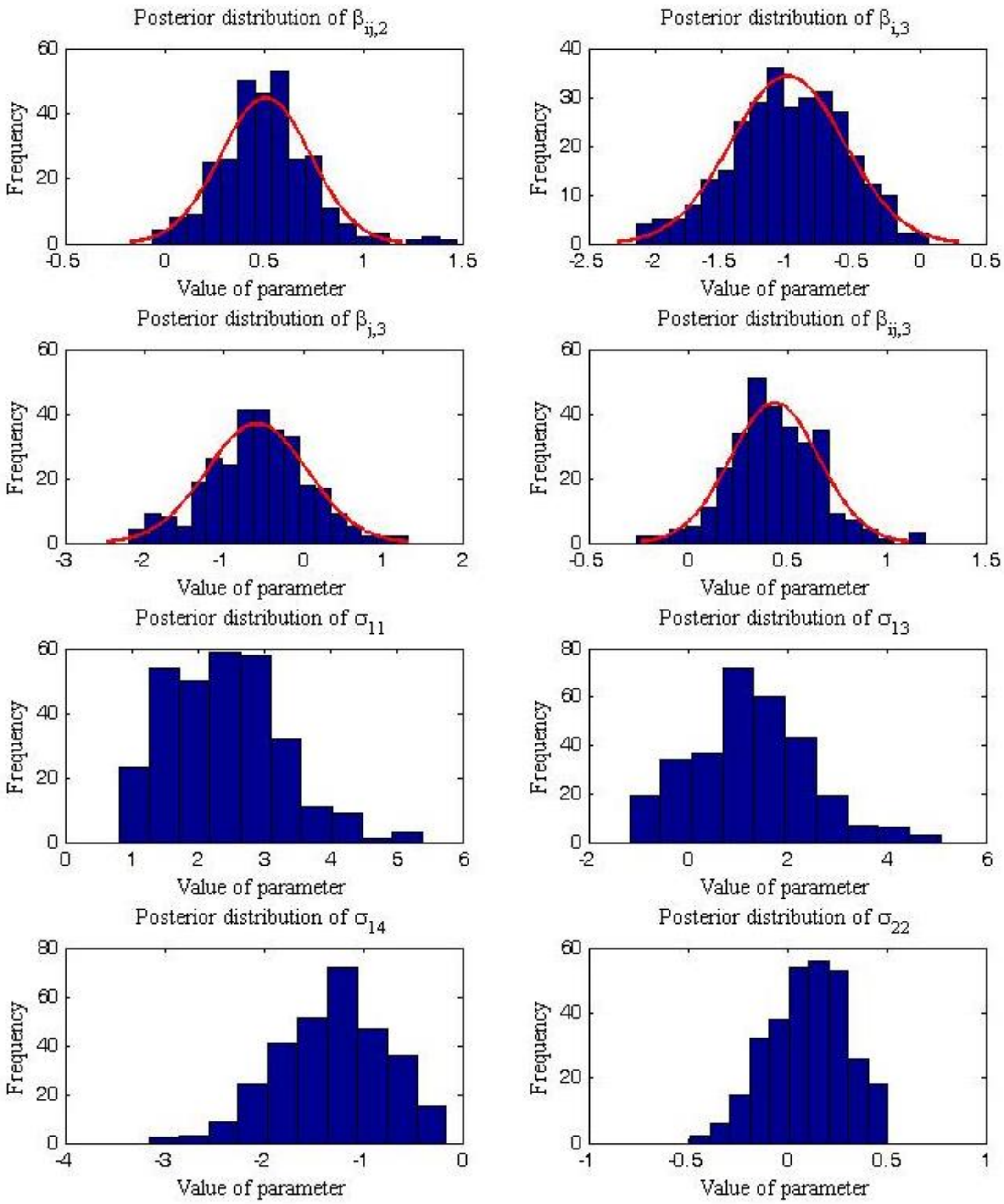


Figure 6.2. Traces of parameters in the carrier-customer collaboration

The traces of the last 300 iterations are stable for all parameters. Therefore, the last 300 iterations are used to derive the posterior distributions. The posterior distributions of estimated parameters are shown in Figure 6.3.


Posterior distribution of α_i
Posterior distribution of α_j
Posterior distribution of α_{ij}
Posterior distribution of $\beta_{i,1}$
Posterior distribution of $\beta_{j,1}$
Posterior distribution of $\beta_{ij,1}$
Posterior distribution of $\beta_{i,2}$
Posterior distribution of $\beta_{j,2}$
Frequency
Value of parameter


Posterior distribution of $\beta_{ij,2}$
Posterior distribution of $\beta_{i,3}$
Posterior distribution of $\beta_{j,3}$
Posterior distribution of $\beta_{ij,3}$
Posterior distribution of σ_{11}
Posterior distribution of σ_{13}
Posterior distribution of σ_{14}
Posterior distribution of σ_{22}
Frequency
Value of parameter

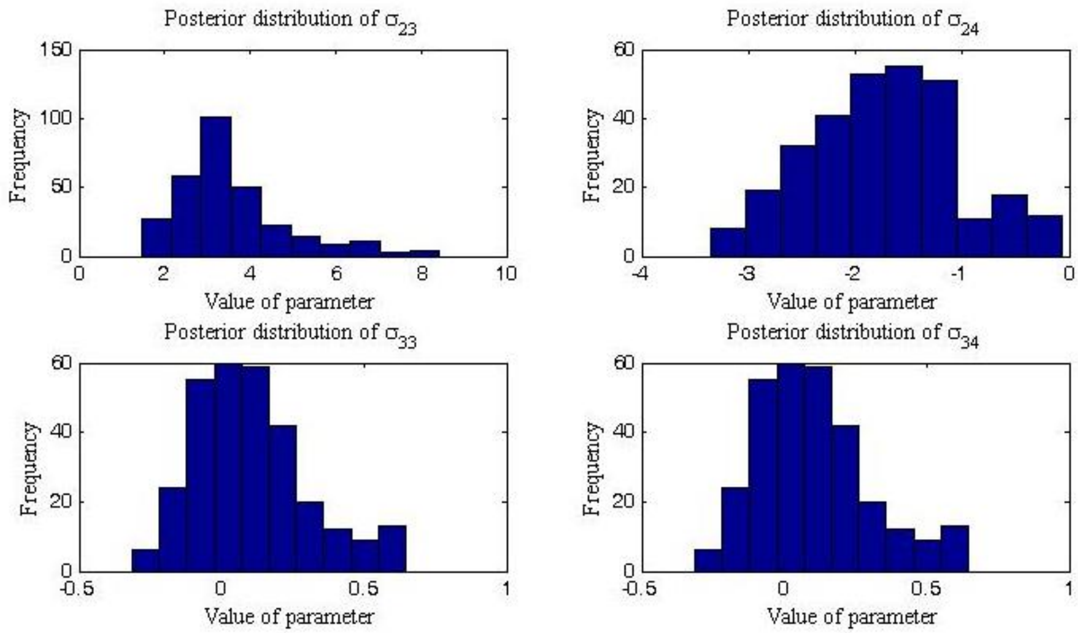


Figure 6.3. Posterior distributions of estimated parameters in the freight carrier-customer collaboration

A summary of the estimation results is reported in Table 6.3.

Table 6.3. Estimation results of carrier-receiver collaboration

Parameters		Mean	S.D.	Pseudo t-stat	95% CI low	95% CI up
Matching Equation						
α_i	FREQ	-0.003	0.063	-0.04	-0.126	0.121
α_j	UPOP	0.724	0.111	6.52	0.506	0.942
α_{ij}	DIST	-0.488	0.034	-14.35	-0.555	-0.421
Joint Response Equation						
Transfer Cost and Reduce Frequency						
$\beta_{i,1}$	SIZE	-0.086	0.248	-0.35	-0.572	0.400
$\beta_{j,1}$	MINC	-2.183	0.961	-2.27	-4.067	-0.299
$\beta_{ij,1}$	INCR	0.509	0.227	2.24	0.064	0.954
Transfer Cost Only						
$\beta_{i,2}$	FARM	-0.997	0.429	-2.32	-1.838	-0.156
$\beta_{j,2}$	MINC	-0.583	0.633	-0.92	-1.824	0.658
$\beta_{ij,2}$	INCR	0.437	0.223	1.96	0.000	0.874
Reduce Frequency Only						
$\beta_{i,3}$	MEAT	0.848	0.681	1.25	-0.487	2.183
$\beta_{j,3}$	MINC	-3.238	0.674	-4.80	-4.559	-1.917
$\beta_{ij,3}$	INCR	0.136	0.258	0.53	-0.370	0.642
Neither Transfer Cost Nor Reduce Frequency				(Base Case)		
σ_{11}		2.399	0.853			
σ_{12}		1.262	1.186			
σ_{13}		-1.31	0.555			
σ_{14}		0.104	0.197			
σ_{22}		3.58	1.298			
σ_{23}		-1.737	0.707			
σ_{24}		0.104	0.197			
σ_{33}		2.144	0.495			

σ_{34}	-0.394	0.153
Number of matched pairs	321	
Number of unmatched pairs	6741	
Log likelihood at null	-12165	
Log likelihood	-11164	
Likelihood Ratio Test	2003	0.000

The likelihood ratio test shows that the fitted model significantly improves the model's goodness-of-fit. Among the independent variables, most of them are statistically significant. These variables provide important and interesting insights into the freight carrier and customer interaction.

In the matching equation, the coefficient of urban population ratio turns out to be positive, indicating that urbanized areas are more attractive to carriers than rural areas. People in urbanized areas are in need of intensive products, leading to a prosperous freight market. Distance has a negative coefficient, indicating that carriers and customers in a short distance are more likely to match with each other.

In the joint response equation, the coefficients of household income at the county level are negative across all equations. Carriers are more likely to consume the hypothetical toll increase by themselves, indicating carriers' market power is relatively low when they are delivering to rich areas. Such a finding may be the result that rich counties have a more competitive freight market. If carriers change their delivery behavior, they would lose existing business. The coefficients of the hypothetical toll increase amount are positive in all equations. If the increased amount is significant, carriers are likely to change behavior. The magnitude of estimated mean values also reveals interesting findings: deploying both options to deal with increased tolls is the most popular option, followed by deploying only cost transfer and then frequency reduction. For the alternative-specific variables, FARM and MEAT are significant in the corresponding equations. The negative coefficient of FARM indicates that carriers working in the farm-related industry sector are less likely to deploy cost transfer. The positive coefficient of MEAT indicates that carriers working in the MEAT industry sector are more likely to deploy frequency reduction.

Most of the estimated coefficients in the variance-covariance matrix are significantly different from zero, verifying the necessity of specifying such a flexible correlation structure. The covariance terms σ_{14}, σ_{24}, and σ_{34} indicate that the process of carrier-customer partner selection affects the choice of toll increase response. The magnitudes of the effects differ across options. If neglecting the effect of partner selection process would result in biased estimation results.

In summary, the proposed joint response model with multinomial outcomes fits the hypothetical toll increase survey data well. The estimated coefficients in the matching equation, joint response equation, and the variance-covariance matrix reveal important explanations of the data generating process. The analysis results improve the understanding of freight agents' interaction and their market power.

7. Conclusions and Future Works

7.1 Conclusions

This dissertation develops an innovative joint response model to address joint decision making of mutually selected decision makers. Many transportation activities are implemented by paired-up agents. For example, public agencies and private companies cooperate in building urban infrastructure. Freight suppliers and customers act jointly in moving cargos from one place to the other. The decision making process by paired up agents had not been given sufficient focus in the research community and it has become even more complicated with the development of information technology in recent decades. Burgeoning technologies enable intensive communication between individuals, leading to frequent matching and joint decision making behavior. For example, the web-based advertisement platform, Craigslist, allows sellers and buyers to seek mutual interests. Provided by the reduced search friction, sellers or buyers could interact with multiple counterparties, leading to an intricate matching network. At any given time, each seller in the matching network is paired up with multiple buyers in the market. This type of matching network is different from traditional analyses of a two-side market in transportation. Traditional analytical frameworks cannot appropriately understand the formation of the emerging matching network and the joint behavior of matched decision makers. The unique feature of the intricate matching network can be attributed to the mutual selection process between agents on the two sides. An agent of one side assesses the characteristics of all potential partners on the other side. If both sides are satisfied with each other, the agents are paired up. If any one side has better options, the collaboration relationship would not be established. From the econometric modeling perspective, models are expected to identify the relationship between a series of explanatory variables. Mutual selection is determined by not only a series of exogenous variables, but the joint decision making of matched agents. If two agents could not make joint decisions, the collaboration relationship would break up. Conversely, the joint decision making is also related to the mutual selection process as joint decisions are only observed between matched agents. Therefore, mutual selection and joint decision making are simultaneous processes, which is an important feature to be captured by the proposed model.

Based on the behavioral background of joint decision making, this dissertation borrows the idea of sample selection models to analyze the problem. The sample selection model consists of two equations with the first equation capturing the mutual selection and the second equation capturing the joint decision making. The first equation defines pairwise utility, a measurement of mutual preference, to characterize the intricate matching network. Based on the observed relationship data (e.g., which agents are matched with each other and which are not), a set of inequality equations are inferred to disentangle the matching network. The second equation uses an ordered probit model and a multinomial probit mode to analyze joint decision outcomes in ordinal and categorical formats. Ordinal and multinomial joint decision outcomes are widely observed in transportation-related activities. A typical joint decision could be shipping frequency/time-of-day in freight transportation, and travel mode/route in passenger transportation. In addition, from a mathematics point of view, ordinal and multinomial outcomes are special cases of the continuous outcome. Finally, the two equations are connected with a flexible variance-covariance matrix to capture their correlation. Their correlation could be identified by empirical data and used to correct the bias resulting from the sample selection process.

Compared with the existing literature, which is noted below in parentheses, the proposed model adds value in investigating a many-to-many matching network (one-to-many matching network in the existing literature), ordinal/multinomial decision outcomes (continuous/binary decision outcomes), and a flexible variance-covariance matrix (restricted variance-covariance matrix).

Given the idea of model frameworks, this dissertation presents the mathematical specification and estimation approach of the proposed model. The estimation approach is a Bayesian MCMC approach with data augmentation. The primary reasons of using such a method are to avoid the optimization of high dimensional integrals in the likelihood function and to take advantage of its flexibility of capturing the correlation between two equations. Then, a series of validation studies are conducted in attempt to show that the estimation approach could recover the parameter values and thus, the model could obtain correct parameter value from an empirical dataset. The parameter traces, posterior distributions, and summary statistics are reported for ordinal and multinomial cases.

Results show that the parameter could be recovered successfully and the proposed model is validated.

The validated model is then applied to analyze empirical data. A flight on-time performance study is first studied considering the matching relationship between airlines and airports. The on-time performance is measured by an ordinal variable with three outcomes. Airline's factor, airport's factor, and their joint factors are analyzed in both matching and joint decision making equations. Results show that their collaboration has significant effects on flight delay, and practitioners may want to consider such an effect in improving flight on-time performance.

Then, a freight survey study is conducted using the proposed model with the multinomial outcome. The survey asks freight carrier's responses to hypothetical toll increases. As freight activity decisions are usually made based on an agreement between multiple agents, carriers' decisions need to consider the influence of other agents. This application focuses on the interaction between freight carriers and customers with an emphasis on the freight carrier's market power. Carrier's factor, customer's factor, and their joint factor are considered in their mutual selection and joint decision making processes. Results show that their matching has significant impacts on the carrier's response to toll increases.

In summary, this dissertation develops an innovative econometric model to fill the void of investigating joint responses of mutually-selected decision makers. The merit of the proposed model is to disentangle the intricate agent matching network, formulate the mutual selection process, and analyze the decision making process in a behaviorally consistent way. The analysis of the behavioral background, mathematical specification, and applications shed light on understanding the behavior of mutually-selected agents, interpreting model results, and providing importation implication for related transportation policies.

7.2 Future Works

Due to the limitations of research time and computation power, some of the work cannot be implemented at this stage, and are left for future research. This section will discuss important potential research directions in the future.

The objective of this dissertation is to understand decision makers' behavior in a simplified two-side market. In practice, markets are much more complicated. A market may consist of multiple sides. For example, a freight supply chain includes suppliers, carriers, customers, and many other decision makers, leading to a chain of two-side pairs or a multi-side partnership. A market may also have a coordinator. Businesses like Airbnb and Uber have a platform where back-end programmers design models to facilitate the matching process. In addition, markets are highly dynamic. A decision maker considers matching with different collaborators at different times. These complexities are not discussed in this dissertation, but the proposed model serves as a foundation. Future works may address these issues by looking into the underlying data generating process and adding additional parameters into the model.

The explanatory variables in the proposed model specification consider factors of both sides, but the error terms do not. The proposed error terms could be further decomposed to capture the supplier's uncaptured effects, customer's uncaptured effects, and joint uncaptured effects. However, the added error terms would complicate the sample selection process and may produce identification issues.

The matching utility is defined for each pair in the proposed model, but may possibly be defined for each decision maker. In the context of this dissertation, pairwise utility is a valid treatment because decision makers of the two sides are assumed to be collaborative. In other situations where decision makers may not be collaborative, defining utility for each decision maker may provide additional insights into understanding the conflicting claims.

The matching equation and the joint decision making equation are connected by the error term in the proposed model. The two equations may be connected in other ways, such as a structural equation where both serve as explanatory variables for each other.

Both applications in the dissertation use a reduced number of explanatory variables to illustrate the use of the proposed model. One reason for using fewer factors is the availability of empirical data. Matching behavior at the disaggregate level is associated with individualized data. Accessing these data often raises confidentiality and safety concerns. Therefore, a possible research direction is to design surveys to collect individualized data.

In summary, the proposed joint response model could be extended in multiple ways and accommodate a lot of possibilities. This dissertation will serve as a foundation for related studies.

Bibliography

Abdulkadiroğlu, A., Pathak, P. A., & Roth, A. E. (2005). The New York City high school match. *American Economic Review*, *95*(2), 364-367.

Adler, J. L., & Blue, V. J. (2002). A cooperative multi-agent transportation management and route guidance system. *Transportation Research Part C: Emerging Technologies*, *10*(5), 433-454.

Albers, S., Koch, B., & Ruff, C. (2005). Strategic alliances between airlines and airports—theoretical assessment and practical evidence. *Journal of Air Transport Management*, *11*(2), 49-58.

Aliabadi, D. E., Kaazemi, A., & Pourghannad, B. (2013). A two-level GA to solve an integrated multi-item supplier selection model. *Applied Mathematics and Computation*, *219*(14), 7600-7615.

Andersson, M., Smith, A., Wikberg, Å., & Wheat, P. (2012). Estimating the marginal cost of railway track renewals using corner solution models. *Transportation Research Part A: Policy and Practice*, *46*(6), 954-964.

Arrow, K. J. (2012). *Social choice and individual values*. New Haven, CT: Yale University Press.

Axelrod, R. M. (1997). *The complexity of cooperation: Agent-based models of competition and collaboration*. Princeton, NJ: Princeton University Press.

Ayala, D., Wolfson, O., Xu, B., DasGupta, B., & Lin, J. (2012). *MATCH-UP 2012: the Second International Workshop on Matching Under Preferences*. Budapest, Hungary: Corvinus University of Budapest.

Bai, R., Li, J., Atkin, J. A., & Kendall, G. (2013). A novel approach to independent taxi scheduling problem based on stable matching. *Journal of the Operational Research Society*, *65*(10), 1501-1510.

Baier, S. L., & Bergstrand, J. H. (2009). Bonus vetus OLS: A simple method for approximating international trade-cost effects using the gravity equation. *Journal of International Economics*, *77*(1), 77-85.

Baïou, M., & Balinski, M. (2002). Erratum: The stable allocation (or ordinal transportation) problem. *Mathematics of Operations Research*, *27*(4), 662-680.

Barbot, C. (2009). Airport and airlines competition: Incentives for vertical collusion. *Transportation Research Part B: Methodological*, *43*(10), 952-965.

Belk, R. (2014). You are what you can access: Sharing and collaborative consumption online. *Journal of Business Research*, *67*(8), 1595-1600.

Bhat, C. R., & Pendyala, R. M. (2005). Modeling intra-household interactions and group decision-making. *Transportation*, *32*(5), 443-448.

Boone, P. (2009). *IATA delay code*. Retrieved April 13, 2016 from http://www.b737mrg.net/downloads/b737mrg_delaycodes.pdf

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management Science*, *51*(1), 30-44.

Chen, J. (2013). Estimation of the loan spread equation with endogenous bank-firm matching. *Structural Econometrics Models, 31(1)*, 251-289.

Choo, E., & Siow, A. (2006). Who marries whom and why. *Journal of Political Economy*, *114*(1), 175-201.

Cohen, B., & Kietzmann, J. (2014). Ride on! Mobility business models for the sharing economy. *Organization & Environment*, *27*(3), 279-296.

Cruijssen, F., Dullaert, W., & Fleuren, H. (2007). Horizontal cooperation in transport and logistics: a literature review. *Transportation Journal*, *46*(3), 22-39.

D'Alfonso, T., & Nastasi, A. (2012). Vertical relations in the air transport industry: a facility-rivalry game. *Transportation Research Part E: Logistics and Transportation Review*, *48*(5), 993-1008.

Davis, H. L. (1976). Decision making within the household. *Journal of Consumer Research*, *2*(4), 241-260.

Disdier, A. C., & Head, K. (2008). The puzzling persistence of the distance effect on bilateral trade. *The Review of Economics and Statistics*, *90*(1), 37-48.

Esmaeili, M., Aryanezhad, M. B., & Zeephongsekul, P. (2009). A game theory approach in seller–buyer supply chain. *European Journal of Operational Research*, *195*(2), 442-448.

Evan, W. M. (1965). Toward a theory of inter-organizational relations. *Management Science*, *11*(10), 1-217.

Fox, J. T. (2008). *Estimating matching games with transfers*. Retrieved April 13, 2016 from http://www.nber.org.libproxy.rpi.edu/papers/w14382.

Fu, X., Homsombat, W., & Oum, T. H. (2011). Airport–airline vertical relationships, their effects and regulatory policy implications. *Journal of Air Transport Management*, *17*(6), 347-353.

Gale, D., & Shapley, L. S. (1962). College admissions and the stability of marriage. *The American Mathematical Monthly*, *69*(1), 9-15.

Goldstein, H. (2011). *Multilevel statistical models*. New York, New York: John Wiley & Sons.

Hamari, J., Sjöklint, M., & Ukkonen, A. (2015). The sharing economy: Why people participate in collaborative consumption. *Journal of the Association for Information Science and Technology*, (June 2015).

He, F., Yin, Y., Chen, Z., & Zhou, J. (2015). Pricing of parking games with atomic players. *Transportation Research Part B: Methodological*, *73*, 1-12.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica*, *47*(1), 153-162.

Helpman, E., Melitz, M., & Rubinstein, Y. (2007). *Estimating trade flows: Trading partners and trading volumes*. Retrieved April 13, 2016 from http://www.nber.org.libproxy.rpi.edu/papers/w12927

Hewings, G. J., Okuyama, Y., & Sonis, M. (2001). Economic interdependence within the Chicago metropolitan area: A Miyazawa analysis. *Journal of Regional Science*, *41*(2), 195-217.

Hihara, K. (2011). An analysis on airport-airline vertical relationships with risk sharing contracts under asymmetric information structures. *Procedia-Social and Behavioral Sciences*, *20*(1), 1090-1098.

Hinsz, V. B., Tindale, R. S., & Vollrath, D. A. (1997). The emerging conceptualization of groups as information processors. *Psychological Bulletin*, *121*(1), 43-64.

Hitsch, G. J., Hortaçsu, A., & Ariely, D. (2010). Matching and sorting in online dating. *The American Economic Review*, *100*(1), 130-163.

Holguín-Veras, J., Aros-Vera, F., & Browne, M. (2015). Agent interactions and the response of supply chains to pricing and incentives. *Economics of Transportation*, *4*(3), 147-155.
Holguin-Veras, J., Pérez, N., Cruz, B., & Polimeni, J. (2006). Effectiveness of financial incentives for off-peak deliveries to restaurants in Manhattan, New York. *Transportation Research Record: Journal of the Transportation Research Board*, *1966*(1), 51-59.
Holguín-Veras, J., Silas, M., Polimeni, J., & Cruz, B. (2008). An investigation on the effectiveness of joint receiver–carrier policies to increase truck traffic in the off-peak hours. *Networks and Spatial Economics*, *8*(4), 327-354.
Holguín-Veras, J., Xu, N., De Jong, G., & Maurer, H. (2011). An experimental economics investigation of shipper-carrier interactions in the choice of mode and shipment size in freight transport. *Networks and Spatial Economics*, *11*(3), 509-532.
Isard, W. (1956). *Location and space-economy*. Cambridge, MA: John Wiley and Sons.
Kamali, A., Ghomi, S. F., & Jolai, F. (2011). A multi-objective quantity discount and joint optimization model for coordination of a single-buyer multi-vendor supply chain. *Computers & Mathematics with Applications*, *62*(8), 3251-3269.
Kheljani, J. G., Ghodsypour, S. H., & O'Brien, C. (2009). Optimizing whole supply chain benefit versus buyer's benefit through supplier selection. *International Journal of Production Economics*, *121*(2), 482-493.
Kita, H. (1999). A merging–giveway interaction model of cars in a merging section: A game theoretic analysis. *Transportation Research Part A: Policy and Practice*, *33*(3), 305-312.
Krajewska, M. A., Kopfer, H., Laporte, G., Ropke, S., & Zaccour, G. (2008). Horizontal cooperation among freight carriers: Request allocation and profit sharing. *Journal of the Operational Research Society, 59*(11), 1483-1491.
Lawler, R. (2012). *Airbnb mobile usage soars as its iOS app passes 1 million downloads, accounts for 26% of all traffic.* Retrieved April 13, 2016 from http://techcrunch.com/2012/09/27/airbnb-1-million-mobile-downloads/
Lawler, R. (2013). *Airbnb tops 10 million guest stays since launch, now has 550,000 properties listed worldwide.* Retrieved April 13, 2016 from http://techcrunch.com/2013/12/19/airbnb-10m/
Leontief, W. W. (1936). Quantitative input and output relations in the economic systems of the United States. *The Review of Economic Statistics*, *18*(3), 105-125.
Leontief, W. W. (1951). *The structure of American economy, 1919-1939: An empirical application of equilibrium analysis*. New York, NY: Oxford University Press.
Levine, A. (2007). *Licensing and scale economies in the biotechnology pharmaceutical industry.* Retrieved April 13, 2016 from http://people.rwj.harvard.edu/~alevine/Licensing_Biotech.pdf
Li, S. X., Huang, Z., Zhu, J., & Chau, P. Y. (2002). Cooperative advertising, game theory and manufacturer–retailer supply chains. *Omega*, *30*(5), 347-357.
Limao, N., & Venables, A. J. (2001). Infrastructure, geographical disadvantage, transport costs, and trade. *The World Bank Economic Review*, *15*(3), 451-479.
Lyft Uber Newsletter. (2016). *Uber prevents drunk driving: advantage over taxis. Lyft Uber Newsletter.* Retrieved April 13, 2016 from http://lyftubernewsletter.com/uber-advantage-over-taxis/.

Macal, C. M., & North, M. J. (2005). Proceedings from the 37th conference on Winter simulation. Orlando, FL: IEEE.

Manheim, M. L. (1979). *Fundamentals of transportation systems analysis; volume 1: Basic concepts*. Cambridge, MA: MIT Press.

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. *Frontiers in Econometrics*, *1*(1), 105-142.

Meixell, M. J., & Norbis, M. (2012). Integrating carrier selection with supplier selection decisions to improve supply chain security. *International Transactions in Operational Research*, *19*(5), 711-732.

Miller, R. E., & Blair, P. D. (2009). *Input-output analysis: Foundations and extensions*. Cambridge, UK: Cambridge University Press.

Mortensen, D. T. (2011). Markets with search friction and the DMP model. *The American Economic Review*, *101*(4), 1073-1091.

Newcomer, E. (2015). *Uber raises funding at $62.5 billion valuation.* Retrieved April 13, 2016 from http://www.bloomberg.com/news/articles/2015-12-03/uber-raises-funding-at-62-5-valuation

Nobile, A. (2000). Comment: Bayesian multinomial probit models with a normalization constraint. *Journal of Econometrics*, *99*(2), 335-345.

Ostrovsky, M. (2008). Stability in supply chain networks. *The American Economic Review*, *98*(1), 897-923.

Paksoy, T., Özceylan, E., & Weber, G. W. (2013). Profit oriented supply chain network optimization. *Central European Journal of Operations Research*, *21*(2), 455-478.

Rashidi, T. H., Auld, J., & Mohammadian, A. K. (2012). A behavioral housing search model: Two-stage hazard-based and multinomial logit approach to choice-set formation and location selection. *Transportation Research Part A: Policy and Practice*, *46*(7), 1097-1107.

Robert, C. P. (1995). Simulation of truncated normal variables. *Statistics and Computing*, *5*(2), 121-125.

Robison, M. H., & Miller, J. R. (1988). Cross-hauling and nonsurvey input-output models: Some lessons from small-area timber economies. *Environment and Planning A*, *20*(11), 1523-1530.

Ronald, N., Arentze, T., & Timmermans, H. (2012). Modeling social interactions between individuals for joint activity scheduling. *Transportation Research Part B: Methodological*, *46*(2), 276-290.

Rose, A. K., & Van Wincoop, E. (2001). National money as a barrier to international trade: The real case for currency union. *The American Economic Review*, *91*(2), 386-390.

Roth, A. E. (1984). The evolution of the labor market for medical interns and residents: A case study in game theory. *The Journal of Political Economy*, *92*(6), 991-1016.

Saraswati, B., & Hanaoka, S. (2014). Airport–airline cooperation under commercial revenue sharing agreements: A network approach. *Transportation Research Part E: Logistics and Transportation Review*, *70*(1), 17-33.

Sen, A. K. (1970). *Collective choice and social welfare*, San Francisco, CA: Holden-Day.

Shapley, L., & Scarf, H. (1974). On cores and indivisibility. *Journal of Mathematical Economics*, *1*(1), 23-37.

Silva, J. S., & Tenreyro, S. (2006). The log of gravity. *The Review of Economics and Statistics*, *88*(4), 641-658.

Siow, A. (2008). How does the marriage market clear? An empirical framework. *Canadian Journal of Economics*, *41*(4), 1121-1155.
Sniezek, J. A., & Henry, R. A. (1990). Revision, weighting, and commitment in consensus group judgment. *Organizational Behavior and Human Decision Processes*, *45*(1), 66-84.
Sørensen, M. (2007). How smart is smart money? A two-sided matching model of venture capital. *The Journal of Finance*, *62*(6), 2725-2762.
Srinivasan, S., & Bhat, C. R. (2005). Modeling household interactions in daily in-home and out-of-home maintenance activity participation. *Transportation*, *32*(5), 523-544.
Sundararajan, A. (2014). *Peer-to-peer businesses and the sharing (collaborative) economy: Overview, economic effects and regulatory issues.* Retrieved April 13, 2016 from http://smbiz.house.gov/uploadedfiles/1-15-2014_revised_sundararajan_testimony.pdf
Swaminathan, J. M., Smith, S. F., & Sadeh, N. M. (1998). Modeling supply chain dynamics: A multiagent approach. *Decision Sciences*, *29*(3), 607-632.
Thaithatkul, P., Seo, T., Kusakabe, T., & Asakura, Y. (2015). A passengers matching problem in ridesharing systems by considering user preference. *Journal of the Eastern Asia Society for Transportation Studies*,*11*(1), 1416-1432.
The Nobel Foundation. (2012). *The Sveriges Riksbank Prize in economic sciences in memory of Alfred Nobel 2012.* Retrieved April 13, 2016 from http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2012/
Thornton, P. R., Shaw, G., & Williams, A. M. (1997). Tourist group holiday decision-making and behaviour: the influence of children. *Tourism Management*, *18*(5), 287-297.
Timmermans, H. J., & Zhang, J. (2009). Modeling household activity travel behavior: Examples of state of the art modeling approaches and research agenda. *Transportation Research Part B: Methodological*, *43*(2), 187-190.
Tinbergen, J. (1962). *Shaping the world economy: suggestions for an international economic policy*. New York, NY: Twentieth Century Fund.
Tindale, R. S., Kameda, T., & Hinsz, V. B. (2003). *Group decision making.* Retrieved April 13, 2016 from http://bs.let.hokudai.ac.jp/~kameda/SageGDMchaptfinal.pdf
U.S. Department of Transportation. (2015). *Moving Ahead for Progress in the 21st Century Act ("MAP-21")*. Washington, DC: Federal Motor Carrier Safety Administration.
U.S. Department of Transportation. (2015). *Air carrier financial reports (form 41 financial data).* Retrieved April 13, 2016 from http://www.transtats.bts.gov/tables.asp?DB_ID=135
U.S. Department of Transportation (2015). *Airline on-time performance data.* Retrieved April 13, 2016 from http://www.transtats.bts.gov/Tables.asp?DB_ID=120
U.S. Department of Transportation (2015). *T-100 domestic market (U.S. carriers).* Retrieved April 13, 2016 from http://www.transtats.bts.gov/Tables.asp?DB_ID=110
Vance, C., & Iovanna, R. (2008). Gender and the automobile: analysis of nonwork service trips. *Transportation Research Record: Journal of the Transportation Research Board*, *2013*(1), 54-61.

Vroom, V. H. (2003). Educating managers for decision making and leadership. *Management Decision*, *41*(10), 968-978.

Walsh, B. (2011). *Today's smart choice: don't own. Share. Times.* Retrieved April 13, 2016 from http://content.time.com/time/specials/packages/article/0,28804,2059521_2059717_2059710,00.html

Wang, X., Agatz, N., & Erera, A. (2014). *Stable matching for dynamic ride-sharing systems. Retrieved* April 13, 2016 from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2611353

Wooldridge, J. M. (2010). *Econometric analysis of cross section and panel data*. Cambridge, MA: MIT Press.

Yang, H., Zhang, A., & Fu, X. (2015). Determinants of airport–airline vertical arrangements: Analytical results and empirical evidence. *Journal of Transport Economics and Policy*, *49*(3), 438-453.

Yao, J. M. (2013). Scheduling optimisation of co-operator selection and task allocation in mass customisation supply chain based on collaborative benefits and risks. *International Journal of Production Research*, *51*(8), 2219-2239.

Zervas, G., Proserpio, D., & Byers, J. (2015). *The rise of the sharing economy: Estimating the impact of Airbnb on the hotel industry*. Retrieved April 13, 2016 from http://papers.ssrn.com/sol3/papers.cfm?abstract-id=2366898

Zhang, A., Fu, X., & Yang, H. G. (2010). Revenue sharing with multiple airlines and airports. *Transportation Research Part B: Methodological*, *44*(8), 944-959.

Zhang, D., Magalhães, D. J. A. V., & Wang, X. C. (2014). Prioritizing bicycle paths in Belo Horizonte City, Brazil: Analysis based on user preferences and willingness considering individual heterogeneity. *Transportation Research Part A: Policy and Practice*, *67*(1), 268-278.

Zhang, D., & Wang, X. (2015a). Investigating the dynamic spillover effects of low-cost airlines on airport airfare through spatio-temporal regression models. *Networks and Spatial Economics*, *8*(4), 1-16.

Zhang, D., & Wang, X. (2015b). Effects of Jobs-Residence Balance on Commuting Patterns: Differences in Employment Sectors and Urban Forms. *Transportation Research Record*, *2500*(1), 80-92.

Zhang, D., & Wang, X. C. (2014). Transit ridership estimation with network Kriging: a case study of Second Avenue Subway, NYC. *Journal of Transport Geography*, *41(1)*, 107-115.

Zhang, J., Kuwano, M., Lee, B., & Fujiwara, A. (2009). Modeling household discrete choice behavior incorporating heterogeneous group decision-making mechanisms. *Transportation Research Part B: Methodological*, *43*(2), 230-250.

Appendix A: MATLAB Code for the Ordinal Case

```
% Bayesian MCMC program for many-to-many with ordered outcomes
% Written by Dapeng Zhang
% 02/09/2016
% zhangdapeng@live.com
%  This m-file has two main parts: generating simulation data and
estimation
% clear all data===================================================
clear all
clc
%% Generate data ===================================================
% Define parameter values =========================================
alpha = [-0.6; 0.9; -0.3]; % parameters in matching equation
beta = [0.3;0.6;-0.9]; % parameters in outcome equation
cut1 = -0.5; cut2 = 0.5; % thresholds in outcome equation
I = 10; J = 100; % The total number of agents in the two-side market
NmatchI_min = 20;  NmatchI_max = 50;
NmatchJ_min = 2;  NmatchJ_max = 5; % allowable number of partners
% Generate the error terms
sig11 = 0.8; sig12 = 0.3; % parameter in the variance-covariance matrix
sig = [sig11  sig12; sig12  1]; % variance-covariance
mu = [0 0]; % Bivariate normal errors
error = mvnrnd(mu,sig,I*J);
epsl = error(:,1); % error term of the 1st outcome equation
ita = error(:,2); % error term of the matching equation
clear error mu sigma

% Generate valuation equations ====================================
DataV = zeros(I*J,6); % column 1: pairwise utility; 2: I; 3 J; 4-6: W
for i = 1:I
    DataV(i*J-(J-1):i*J,2) = i*ones(J,1); % I
    DataV(i*J-(J-1):i*J,3) = 1:J; % J
end
C_I = zeros(I,2); C_I(:,1) = 1:I; C_I(:,2) = randn(I,1); % I's factor
C_J = zeros(J,2); C_J(:,1) = 1:J; C_J(:,2) = randn(J,1); % J's factor
for ij = 1:I*J
    row1 = DataV(ij,2); row2 = DataV(ij,3);
```

```
    DataV(ij,4) = C_I(row1,2);
    DataV(ij,5) = C_J(row2,2);
end
DataV(:,6) = randn(I*J,1); % I-J factor
DataV(:,1) = DataV(:,4:6) * alpha + ita;
clear C_I C_J i ij row1 row2

% Matching structure ==================================================
tmp = DataV;
quotaI = zeros(I,1); quotaJ = zeros(J,1); n = 0; minquotaI = 0; minquotaJ
= 0;
while  (minquotaI  <  NmatchI_min  ||  minquotaJ  <  NmatchJ_min)  &&
isempty(DataV) == 0
    row1 = find(DataV(:,1) == max(DataV(:,1)));
    i = DataV(row1, 2); j = DataV(row1,3);
    if quotaI(i) >= NmatchI_max || quotaJ(j) >= NmatchJ_max
    else
    quotaI(i) = quotaI(i) + 1;
    quotaJ(j) = quotaJ(j) + 1;
    n = n + 1;
    match(n,:) = DataV(row1,:); % matching pair index output
    end
    DataV(row1,:) = [];
    minquotaI = min(quotaI); minquotaJ = min(quotaJ);
end
Nmatch = length(match); % Number of matched agents
DataV = tmp;
clear tmp

% Generate outcome equations ==========================================
DataO = zeros(Nmatch,6); % only observed 1 outcome 2-3 index 4-6 W
match = sortrows(match, [2 3]);
DataO(:,2:3) = match(:,2:3); % column 2-3 i,j index
rowm = zeros(Nmatch,1);
for ij = 1 : Nmatch
    i = DataO(ij,2); j = DataO(ij,3);
    rowm(ij,1) = find(DataV(:,2) == i & DataV(:,3) == j);
    DataO(ij, 4:6) = DataV(rowm(ij,1), 4:6); % colume 4-6
```

```
end
Y_star = DataO(:,4:6) * beta + epsl(rowm);
Y(Y_star < cut1) = 1; % cat 1
Y(Y_star < cut2 & Y_star > cut1) = 2; % cat 2
Y(Y_star > cut2) = 3; % cat 3
DataO(:, 1) = Y;

clear alpha beta epsl NmatchI_max NmatchI_min NmatchJ_max NmatchJ_min i
clear ij ita j match minquotaI minquotaJ n quotaI quotaJ row1 sig sig11
clear sig12 Y_star

%% Estimation ==========================================================
% Define the initial values of estimated parameters ===================
beta = [0;0;0]; inibeta = beta;
alpha = [-0.1;0.1;-0.1]; inialpha = alpha; % use vary small values
sig11 = 1; sig12 = 0; sig = [sig11 sig12; sig12 1];
cut1 = -0.5; cut2 = 0.25;
cut = [-inf; cut1; cut2; inf];

% Use simpler notations of data =======================================
DataVm = DataV(rowm,:); DataVc = DataV; DataVc(rowm,:) = [];
DataVm_rec = DataVm; DataVc_rec = DataVc;
W = DataV(:,4:6); Wm = DataV(rowm, 4:6); Wc = W; Wc(rowm,:) = [];
Vm = Wm*alpha; Vc = Wc*alpha;
X = DataO(:,4:6); Y = DataO(:,1); Y_star = X*beta;
clear V W

% Estimation using Bayesian MCMC ======================================
inisigalpha = [10 0 0
            0 10 0
            0 0 10];
inisigbeta = [10 0 0
            0 10 0
            0 0 10];

% Estimation Iteration parameters and save the results ================
MaxIter = 5000;
```

```
result_alpha = zeros(3,MaxIter); result_alpha(:,1) = alpha;
result_beta = zeros(3,MaxIter); result_beta(:,1) = beta;
result_sig = zeros(2,MaxIter); result_sig(:,1) = [sig11;sig12];
result_cut = zeros(2,MaxIter); result_cut(:,1) = [cut1;cut2];

% Iteration starts =====================================================
for iter = 2:MaxIter
% update alpha
M_alpha = inv(inisigalpha) + Wc' * Wc + (1-sig12^2/sig11) \ (Wm' * Wm);
N_alpha = inisigalpha\inialpha + Wc'*Vc + (1-sig12^2/sig11) \ Wm'*(Vm-
sig12/sig11*(Y_star-X*beta));
alpha = mvnrnd(M_alpha\N_alpha, inv(M_alpha))';
result_alpha(1:3,iter) = alpha;
clear M_alpha N_alpha

% update beta
M_beta = inv(inisigbeta) + X' * inv(sig11 - sig12^2) * X;
N_beta = inisigbeta\inibeta + X' * inv(sig11 - sig12^2) * (Y_star -
sig12*(Vm - Wm*alpha));
beta = mvnrnd(M_beta\N_beta, inv(M_beta))';
result_beta(1:3,iter) = beta;
clear M_beta N_beta

% update matching value
meanVc = Wc * alpha;
updateVc = zeros(length(meanVc),1);
for ij = 1: length(meanVc)
    i = DataVc(ij, 2);
    row1 = find(DataVm(:,2) == i);
    V_up_2 = min(Vm(row1,1));
    j = DataVc(ij, 3);
    row2 = find(DataVm(:,3) == j);
    V_up_1 = min(Vm(row2,1));
    V_up = max(V_up_1, V_up_2);
    updateVc(ij,1) = rmvnrnd(meanVc(ij),1,1,1,V_up);
end
DataVc(:,1) = updateVc;
```

```
Vc = DataVc(:,1);
clear meanVc updateVc ij i row1 V_up_2 j row2 V_up_1 V_up updateVc

meanVm = Wm * alpha + sig12/sig11*(Y_star-X*beta);
update_V = zeros(Nmatch,1);
for ij = 1:Nmatch
    i = DataVm(ij,2);
    row1 = find(DataVc(:,2) == i);
    V_ij = DataVc(row1,1:3); % all value of ij
    for aa = 1:length(row1)
        jj = V_ij(aa,3);    % jj represents each j
        row2 = find(DataVm(:,3) == jj);
        V_ij(aa,4) = min(Vm(row2,1)); % matched value of j
        if V_ij(aa,1) > V_ij(aa,4)
            V_ij(aa,5) = V_ij(aa,1);
        else
            V_ij(aa,5) = -inf;
        end
    end
    V_low_2 = max(V_ij(:,5)); % maybe empty

    j = DataVm(ij,3);
    row3 = find(DataVc(:,3) == j);
    V_ij = DataVc(row3,1:3);
    for aa = 1:length(row3)
        ii = V_ij(aa,2);
        row4 = find(DataVm(:,2) == ii);
        V_ij(aa,4) = min(Vm(row4,1));
        if V_ij(aa,1) > V_ij(aa,4)
            V_ij(aa,5) = V_ij(aa,1);
        else
            V_ij(aa,5) = -inf;
        end
    end
    V_low_1 = max(V_ij(:,5)); % maybe empty
    V_low = max(V_low_1, V_low_2);
    if V_low > -inf
```

```
        update_V(ij,1)     =     rmvnrnd(meanVm(ij),1-sig12^2/sig11,1,-1,-
V_low);
    else
        update_V(ij,1) = mvnrnd(meanVm(ij),1-sig12^2/sig11);
    end
end

DataVm(:,1) = update_V;
Vm = DataVm(:,1);
clear meanVm update_V ij i row1 V_ij aa jj row2 V_low_2 j row3 ii row4
clear V_low_1 V_low update_V

% update latent variables in the outcome equation
meanY_star = X * beta + sig12*(Vm - Wm * alpha);
for i = 1:Nmatch
    j = Y(i,1);
    Y_star(i,1)     =     rmvnrnd(meanY_star(i),sig11-sig12^2,1,[1;-     …
1],[cut(j+1);-cut(j)]);
end
cut2_min = max(max(Y_star(Y == 2)), cut1);
cut2_max = min(min(Y_star(Y == 3)), 0.8);
cut2 = unifrnd(cut2_min, cut2_max);
cut = [-inf cut1 cut2 inf];
result_cut(2,iter) = cut2;

% update sig
tmp6 = zeros(2,2);
for ij = 1:Nmatch
    ita = Vm(ij) - Wm(ij,:) * alpha;
    epsl = Y_star(ij) - X(ij,:) * beta;
    tmp5 = [epsl; ita] * [epsl; ita]';
    tmp6 = tmp6 + tmp5;
end
VV = 3*eye(2) + tmp6;
L = chol(inv(VV),'lower');
A = zeros(2,2);
A(1,1) = sqrt(chi2rnd(Nmatch+3+1-1));
```

```
A(2,2) = inv(sqrt(1)*L(2,2));
A(2,1) = normrnd(0,1);
sig = inv(L)' * inv(A)' * inv(A) * inv(L);
sig11 = sig(1,1); sig12 = sig(1,2);
result_sig(1:2,iter) = [sig(1,1);sig(1,2)];
clear tmp6 ita epsl tmp5 tmp6 VV L A ij
end

% plot trace ==========================================================
figure
subplot(4,3,1)
plot(result_alpha(1,1:iter))
subplot(4,3,2)
plot(result_alpha(2,1:iter))
subplot(4,3,3)
plot(result_alpha(3,1:iter))
subplot(4,3,4)
plot(result_beta(1,1:iter))
subplot(4,3,5)
plot(result_beta(2,1:iter))
subplot(4,3,6)
plot(result_beta(3,1:iter))
subplot(4,3,7)
plot(result_sig(1,1:iter))
subplot(4,3,8)
plot(result_sig(2,1:iter))
subplot(4,3,10)
plot(result_cut(1,1:iter))
subplot(4,3,11)
plot(result_cut(2,1:iter))
drawnow
```

Appendix B: MATLAB Code for the Multinomial Case

```
% Bayesian MCMC program for many-to-many with multinomial outcomes
% Parameters are similar as those in the ordinal case
clear all
clc
%% Generate data =====================================================
P = 2; % number of choices
alpha = [-0.6; 0.9; -0.3];
beta1 = [-0.9; 0.6; 0.3];
beta2 = [0.6; 0.3; -0.9];
I = 50; J = 50; % total number of agents
NmatchI_min = 25;  NmatchI_max = 25;
NmatchJ_min = 25;  NmatchJ_max = 25;
% Generate the error terms
sig11 = 1.5; sig12 = -0.1; sig13 = -0.2; sig22 = 0.6; sig23 = -0.3;
sig = [sig11 sig12 sig13; sig12 sig22 sig23; sig13 sig23  1];
mu = [0 0 0];
error = mvnrnd(mu,sig,I*J);
ita = error(:,3);
epsl1 = error(:,1);
epsl2 = error(:,2);
clear error mu sig11 sig12 sig13 sig22 sig23

% Generate valuation equations
DataV = zeros(I*J,6);
for i = 1:I
    DataV(i*J-(J-1):i*J,2) = i*ones(J,1);
    DataV(i*J-(J-1):i*J,3) = 1:J;
end
C_I = zeros(I,2); C_I(:,1) = 1:I; C_I(:,2) = randn(I,1);
C_J = zeros(J,2); C_J(:,1) = 1:J; C_J(:,2) = randn(J,1);
for ij = 1:I*J
    row1 = DataV(ij,2); row2 = DataV(ij,3);
    DataV(ij,4) = C_I(row1,2);
    DataV(ij,5) = C_J(row2,2);
end
DataV(:,6) = randn(I*J,1);
```

```
DataV(:,1) = DataV(:,4:6) * alpha + ita;
clear C_I C_J i ij row1 row2

% Matching determination
tmp = DataV;
quotaI = zeros(I,1); quotaJ = zeros(J,1); n = 0; minquotaI = 0; minquotaJ
= 0;
while  (minquotaI  <  NmatchI_min  ||  minquotaJ  <  NmatchJ_min)  &&
isempty(DataV) == 0
    row1 = find(DataV(:,1) == max(DataV(:,1)));
    i = DataV(row1, 2); j = DataV(row1,3);
    if quotaI(i) >= NmatchI_max || quotaJ(j) >= NmatchJ_max
    else
    quotaI(i) = quotaI(i) + 1;
    quotaJ(j) = quotaJ(j) + 1;
    n = n + 1;
    match(n,:) = DataV(row1,:);
    end
    DataV(row1,:) = [];
    minquotaI = min(quotaI); minquotaJ = min(quotaJ);
end
Nmatch = length(match);
DataV = tmp;
clear tmp

% Generate outcome equations
DataO1 = zeros(Nmatch,6); DataO2 = zeros(Nmatch,6);
match = sortrows(match, [2 3]);
DataO1(:,2:3) = match(:,2:3); DataO2(:,2:3) = match(:,2:3);
rowm = zeros(Nmatch,1);
for ij = 1 : Nmatch
    i = DataO1(ij,2); j = DataO1(ij,3);
    rowm(ij,1) = find(DataV(:,2) == i & DataV(:,3) == j);
    DataO1(ij, 4:6) = DataV(rowm(ij,1), 4:6);
    DataO2(ij, 4:6) = DataV(rowm(ij,1), 4:6);
end
Y1_star = DataO1(:,4:6) * beta1 + epsl1(rowm);
Y2_star = DataO2(:,4:6) * beta2 + epsl2(rowm);
```

```
Y = zeros(Nmatch,1);
for i = 1:Nmatch
    Y_star = [Y1_star(i);Y2_star(i)];
    [row,~] = find(Y_star == max([Y_star;0]));
    if row == 1
        Y(i,1) = 1;
    elseif row == 2;
        Y(i,1) = 2;
    else
        Y(i,1) = 3;
    end
end
clear alpha beta1 beta2 epsl1 epsl2 Y_star Y1_star Y2_star
clear NmatchI_max NmatchI_min NmatchJ_max NmatchJ_min i ij ita j
clear match minquotaI minquotaJ n quotaI quotaJ row row1 sig sig11 sig12

%% Estimation ========================================================
% Define the initial values of estimated parameters ==================
beta1 = [0;0;0]; beta2 = [0;0;0]; inibeta1 = beta1; inibeta2 = beta2;
alpha = [-0.5;0.5;-0.5]; inialpha = alpha;
sig11 = 1; sig12 = 0; sig13 = 0; sig22 = 1; sig23 = 0;
sig = [sig11 sig12 sig13; sig12 sig22 sig23; sig13 sig23 1];

% Use simpler notations of data
DataVm = DataV(rowm,:); DataVc = DataV; DataVc(rowm,:) = [];
DataVm_rec = DataVm; DataVc_rec = DataVc;
W = DataV(:,4:6); Wm = DataV(rowm, 4:6); Wc = W; Wc(rowm,:) = [];
Vm = Wm*alpha; Vc = Wc*alpha;
X1 = DataO1(:,4:6); X2 = DataO2(:,4:6);
Y1_star = Y1_star_rec; Y2_star = Y2_star_rec;
DataVm_rec = DataVm; DataVc_rec = DataVc;

inisigalpha = [10 0 0
              0 10 0
              0 0 10];
inisigbeta1 = [10 0 0
```

```
            0 10 0
            0 0 10];
inisigbeta2 = [10 0 0
            0 10 0
            0 0 10];
% Estimation Iteration parameters
result_alpha = zeros(3, MaxIter); result_alpha(:,1) = inialpha;
result_beta = zeros(6, MaxIter); result_beta(1:3,1) = inibeta1;
result_beta(4:6,1) = inibeta2; result_sig = zeros(5, MaxIter);
result_sig(:,1) = [sig11;sig12;sig13;sig22;sig23];

for iter = 2:MaxIter
    waitbar(iter/MaxIter)
% update alpha
sigita = 1-[sig13 sig23]*inv([sig11 sig12;sig12 sig22])*[sig13;sig23];
M_alpha = inv(inisigalpha) + Wc'* Wc + inv(sigita)*Wm' * Wm;
N_alpha = inisigalpha\inialpha + Wc'*Vc ;
for ij = 1:Nmatch
    N_alpha_tmp = inv(sigita)*Wm(ij,:)'...
        *(Vm(ij)-[sig13 sig23]*...
        inv([sig11 sig12;sig12 sig22])*...
        [Y1_star(ij)-X1(ij,:)*beta1; Y2_star(ij)-X2(ij,:)*beta2]);
    N_alpha = N_alpha + N_alpha_tmp;
end
alpha = mvnrnd(M_alpha\N_alpha, inv(M_alpha))';
result_alpha(1:3,iter) = alpha;
clear sigita M_alpha N_alpha N_alpham N_alpha_tmp

% update beta
M_beta = inv(inisigbeta1); N_beta = inv(inisigbeta1)*inibeta1;
sigepsl1    =    sig11    -    [sig12    sig13]*inv([sig22    sig23;sig23
1])*[sig12;sig13];
for ij = 1:Nmatch
    M_beta_tmp =  inv(sigepsl1) * X1(ij,:)' * X1(ij,:);
    M_beta = M_beta + M_beta_tmp;
    N_beta_tmp =  inv(sigepsl1) * X1(ij,:)'*...
        (Y1_star(ij) - [sig12 sig13]*inv([sig22 sig23; sig23 1])*...
        [Y2_star(ij)-X2(ij,:)*beta2; Vm(ij)-Wm(ij,:)*alpha]);
```

```
    N_beta = N_beta + N_beta_tmp;
end
beta1 = mvnrnd(M_beta\N_beta, inv(M_beta))';
result_beta(1:3,iter) = beta1;
clear M_beta M_beta_tmp N_beta N_beta_tmp

M_beta = inv(inisigbeta2); N_beta = inv(inisigbeta2)*inibeta2;
sigepsl2 = sig22 - [sig12 sig23] * inv([sig11 sig13; sig13
1])*[sig12;sig23];
for ij = 1:Nmatch
    M_beta_tmp = inv(sigepsl2) * X2(ij,:)' * X2(ij,:);
    M_beta = M_beta + M_beta_tmp;
    N_beta_tmp = inv(sigepsl2) * X2(ij,:)'...
        *(Y2_star(ij) - [sig12 sig23]*inv([sig11 sig13; sig13 1])*...
        [Y1_star(ij)-X1(ij,:)*beta1; Vm(ij)-Wm(ij,:)*alpha]);
    N_beta = N_beta + N_beta_tmp;
end
beta2 = mvnrnd(M_beta\N_beta, inv(M_beta))';
result_beta(4:6,iter) = beta2;
clear M_beta M_beta_tmp N_beta N_beta_tmp

% update matching value
meanVc = Wc * alpha;
updateVc = zeros(length(meanVc),1);
for ij = 1: length(meanVc)
    i = DataVc(ij, 2);
    row1 = find(DataVm(:,2) == i);
    V_up_2 = min(Vm(row1,1));
    j = DataVc(ij, 3);
    row2 = find(DataVm(:,3) == j);
    V_up_1 = min(Vm(row2,1));
    V_up = max(V_up_1, V_up_2);
    updateVc(ij,1) = rmvnrnd(meanVc(ij),1,1,1,V_up);

end
DataVc(:,1) = updateVc;
Vc = DataVc(:,1);
```

```
clear meanVc updateVc ij i row1 V_up_2 j row2 V_up_1 V_up updateVc

meanVm = zeros(Nmatch,1);
update_V = zeros(Nmatch,1);
for ij = 1:Nmatch
    meanVm(ij,1) = Wm(ij,:) * alpha + [sig13 sig23]*([sig11 sig12; sig12
sig22]\[Y1_star(ij)-X1(ij,:)*beta1; Y2_star(ij)-X2(ij,:)*beta2]);
end
for ij = 1:Nmatch
    i = DataVm(ij,2);
    row1 = find(DataVc(:,2) == i);
    V_ij = DataVc(row1,1:3);
    for aa = 1:length(row1)
        jj = V_ij(aa,3);
        row2 = find(DataVm(:,3) == jj);
        V_ij(aa,4) = min(Vm(row2,1));
        if V_ij(aa,1) > V_ij(aa,4)
            V_ij(aa,5) = V_ij(aa,1);
        else
            V_ij(aa,5) = -inf;
        end
    end
    V_low_2 = max(V_ij(:,5));

    j = DataVm(ij,3);
    row3 = find(DataVc(:,3) == j);
    V_ij = DataVc(row3,1:3);
    for aa = 1:length(row3)
        ii = V_ij(aa,2);
        row4 = find(DataVm(:,2) == ii);
        V_ij(aa,4) = min(Vm(row4,1));
        if V_ij(aa,1) > V_ij(aa,4)
            V_ij(aa,5) = V_ij(aa,1);
        else
            V_ij(aa,5) = -inf;
        end
    end
    V_low_1 = max(V_ij(:,5));
```

```
    V_low = max(V_low_1, V_low_2);
    sigb = 1 - [sig13 sig23] * ([sig11 sig12; sig12 sig22]\[sig13;
sig23]);
    if V_low > -inf
        update_V(ij,1) = rmvnrnd(meanVm(ij),sigb,1,-1,-V_low);
    else
        update_V(ij,1) = mvnrnd(meanVm(ij),sigb);
    end
end

DataVm(:,1) = update_V;
DataV_result(:,iter) = update_V;
Vm = DataVm(:,1);
clear V_low V_low1 V_low2

% update sig
tmp = zeros(3,3);
for ij = 1:Nmatch
    error = [Y1_star(ij)-X1(ij,:)*beta1;...
        Y2_star(ij)-X2(ij,:)*beta2;...
        Vm(ij)-Wm(ij,:)*alpha];
    tmp1 = error * error';
    tmp = tmp + tmp1;
end
VV = 3*eye(3) + tmp;
L = chol(inv(VV),'lower');
A = zeros(3,3);
A(1,1) = sqrt(chi2rnd(Nmatch+3+1-1));
A(2,2) = sqrt(chi2rnd(Nmatch+3+1-2));
A(3,3) = inv(sqrt(1)*L(3,3));
A(2,1) = normrnd(0,1);A(3,1) = normrnd(0,1);A(3,2) = normrnd(0,1);
sig = inv(L)' * inv(A)' * inv(A) * inv(L);
sig11 = sig(1,1); sig12 = sig(1,2); sig13 = sig(1,3);
sig22 = sig(2,2); sig23 = sig(2,3);
result_sig(1:5,iter) = [sig(1,1);sig(1,2); sig(1,3); sig(2,2); sig(2,3)];
clear tmp error tmp1 VV L A
```

```
% update latent variables in the outcome equation
sigc = [sig11 sig12; sig12 sig22] - [sig13;sig23]*[sig13 sig23];
for ij = 1:Nmatch
    mu = [X1(ij,:)*beta1; X2(ij,:)*beta2] + ...
        [sig13;sig23]*(Vm(ij)-Wm(ij,:)*alpha);
    if Y(ij) == 1
        tmp = rmvnrnd(mu,sigc,1,[-1 1; -1 0], [0; 0])';
        Y1_star(ij) = tmp(1); Y2_star(ij) = tmp(2);
    elseif Y(ij) == 2
        tmp = rmvnrnd(mu,sigc,1,[1 -1; 0 -1], [0; 0])';
        Y1_star(ij) = tmp(1); Y2_star(ij) = tmp(2);
    elseif Y(ij) == 3
        tmp = rmvnrnd(mu,sigc,1,[1 0; 0 1], [0; 0])';
        Y1_star(ij) = tmp(1); Y2_star(ij) = tmp(2);
    end
end
Y1_star = Y1_star_rec; Y2_star = Y2_star_rec;
clear error sigc ij tmp
end

figure
subplot(5,3,1)
plot(result_alpha(1,1:iter))
subplot(5,3,2)
plot(result_alpha(2,1:iter))
subplot(5,3,3)
plot(result_alpha(3,1:iter))
subplot(5,3,4)
plot(result_beta(1,1:iter))
subplot(5,3,5)
plot(result_beta(2,1:iter))
subplot(5,3,6)
plot(result_beta(3,1:iter))
subplot(5,3,7)
plot(result_beta(4,1:iter))
subplot(5,3,8)
plot(result_beta(5,1:iter))
subplot(5,3,9)
```

```
plot(result_beta(6,1:iter))
subplot(5,3,10)
plot(result_sig(1,1:iter))
subplot(5,3,11)
plot(result_sig(2,1:iter))
subplot(5,3,12)
plot(result_sig(3,1:iter))
subplot(5,3,13)
plot(result_sig(4,1:iter))
subplot(5,3,14)
plot(result_sig(5,1:iter))
```